Problem Solving Exercise

Background:
This document is an elaboration of an exercise used once in FYF101 many years ago. The point is to think about the nature of problems themselves. In your earlier academic endeavors, very likely problems were given to you, and every problem had one and only one right answer. Suppose a problem has more than one right answer? Or no right answer? If you are taking a multiple-choice test in school, you'd be outraged. "None of these choices are right!" And then, "It's not fair!" When the whole purpose of a test question is to slap a grade on you, yes, this is a problem. But, real world problems have a habit of being "messy." This is a little exercise in looking at examples along the spectrum of kinds of problems. First, I'll present the "Questions." Take a look at that as an exercise. Then, I have some commentary on the exercise, which is what was talked about afterwards in the class when I used this exercise.

The Exercise:
As a group, solve the following problems. Think about what you need to do in order to solve each problem, and whether there are alternative ways of viewing or solving the problem.

1. What number is spelled "seven"?

2. In what nation was Einstein born?

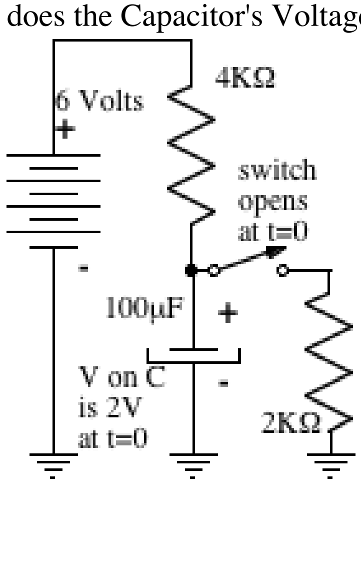
3. Find the sum: 25 + 3 =

4 Convert the Binary number 10010101 to decimal:

5. A current of 1 Ampere flows through a 200 Ohm resistor. What is the Voltage across it?

6 A resistor has a voltage of 4 Volts across it, and conducts a current of 2 mA. Find the resistance.

7. A capacitor the circuit below starts to charge when it's Voltage across it is 2 Volts. How fast does the Capacitor's Voltage change at first (in Volts per second)?



8. How can your sound project circuit be improved? (The "sound project" was a digital circuit using a counter and a Read Only Memory, a D/A converter and an amplifier that could, when a button was pushed, produce an "interesting" sound. You could read this question as, "design a project that does this.")

9. What should be done to solve the problem of poverty? (Use back if more space needed.)

Discussion:

Let's take a look at these one at a time. The problems range from easy to hard. But, more than that, they require differ kinds of solution techniques.

1. What number is spelled "seven"?

This is a reading test. Do you understand the relationship between the five-letter word "seven" and the number 7 (and do you understand what a number is?) Most people have this down about first grade. It's not something you even have to think about, you just automatically know it. It comes with the mental equipment of being human and having had a reasonably decent childhood.

2. In what nation was Einstein born?

This is a question of fact. Do you know that there was a fellow named Einstein, and that he was born in Germany? (Yes, you could cite that there are others named Einstein who are not the famous physicist, like the bagels guy, but when a name is presented like this, with no additional qualifiers, you should think, who is the most important person with that name? So, it's not quite just knowing a fact, but somewhat an issue of being culturally informed.) Not everybody knows that Einstein was born in Germany. Or that he fled Germany ahead of World War II, or why he was well advised to do so. But, somewhere in school you should have learned this.

3. Find the sum: 25 + 3 =

While this could be looked at as a fact question, when students memorize addition tables they normally do it only for single digits. Adding 25 to 3 requires, beyond that, a procedure to do arithmetic for numbers having multiple digits. You would normally learn to do this in 3rd or 4th grade, if not sooner. What you learned would be generally applicable. You would also know how, similarly, to add 25 + 4. (To do 25 + 5 is a bit more complicated, but you would have learned how to do that at the same time too.) This is a step beyond just knowing facts, as in the previous question. There are also procedural processes, like drawing inferences, that are applicable to the humanities as well as mathematics.

4 Convert the Binary number 10010101 to decimal:

This is also a procedural question. But, as presented, it has more than one answer. Three possible answers include 149, -21, and -107. How could there be multiple answers to so simple a question? Because the problem did not state explicitly what the binary representation is. It could be "unsigned" (149) in which case the initial 1 represents 128. It could be -21 if the representation is that the first 1 represents a minus sign. This is the "sign-magnitude" representation. It would be -107 using the "two's complement" method for representing negative numbers that's overwhelmingly common in today's computers. So, here's an example where you really don't have enough information to solve the problem. You need to either make an assumption or ask what is meant. Many, many problems are like this. As an engineer, you need to guard against the possibility that your client is thinking one way and you are assuming another. Some problems have "no solution": How to represent the numbers 0 to 256 with 8 bits.

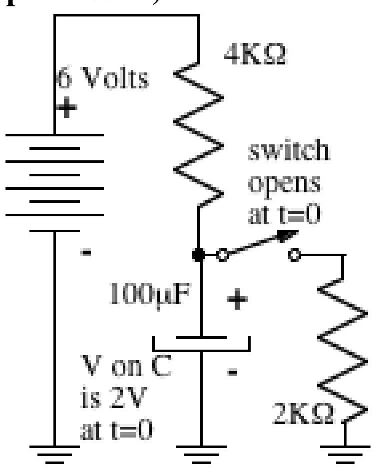
5. A current of 1 Ampere flows through a 200 Ohm resistor. What is the Voltage across it?

Here is an example of a math "word problem" that is easily solved using a common and easily applied formula, Ohm's Law, which states: V=IR. V is Voltage (in Volts), I is current in Amperes, and R is resistance in Volts per Ampere, called Ohms. To solve the problem, just replace the symbols on the right-hand side of the equal mark with the corresponding numbers, do a calculation, and that's the answer in Volts. Easy, right? In this case, yes. But, suppose someone told you the current was 1000 mA (milliAmperes)? You have to worry about units then. Keeping track of units is very important in engineering. This formula works ONLY because you can substitute Volts per Ampere for Ohms, then cancel Amperes per Ampere to leave just Volts.

6 A resistor has a voltage of 4 Volts across it, and conducts a current of 2 mA. Find the resistance.

This is very similar to the above problem, but there's an extra element of complexity. You have Voltage (the left-hand side of V=IR) and current (one factor on the right-hand side). Plugging in, you get 4 Volts = 2 mA • R. What to do? To solve this, you need to resort to Algebra. You know the algebraic principle that you can divide both sides by 2 mA to get: 4 Volts / 2 mA = R. You can rewrite this as R = 4 Volts / 2 mA. You can then perform the calculation on just the right-hand side to get R = 2 V/mA. Knowing that m= .001, and that an Ohm is 1Volt/A, you can convert this into R = 2K Ohms (or R=2000 Ohms). So, here, you are applying a formula, but it does not come as a ready-made fit for the problem as stated. You must use mathematics, algebra in this case, as well as the factual knowledge of the formula, and unit cancellation and conversion, to solve the problem. Doing this kind of thing, and much more complex versions of it, is routine in the world of engineering.

7. (Problem 11 from the test) A capacitor the circuit below starts to charge when it's Voltage across it is 2 Volts. How fast does the Capacitor's Voltage change at first (in Volts per second)?



This is an example of a problem that requires more than just a formula. There are many kinds of similar problems that could be constructed. This is about a simple as it gets. Several steps are needed. First, the person trying to solve the problem needs to recognize the symbols used, and how the connections and components represented correspond to the real-world. It's like reading a map. The symbols used on a map may bear a resemblance to the real-world objects, but are simplified, reduced to two dimensions, and scaled to a convenient size. The correspondence to the reality must be learned.

In this case, a battery (Positive Voltage source) connects through a resistor to a capacitor. Both are "grounded". The idea of ground, like "sea level" on a map, establishes a reference point for the symbolic representation of reality. There is a switch that connects the capacitor also through a resistor to ground. So, there is a temporal, or time-based component, to the representation. When the switch is open, the capacitor charges. When the switch closes, the capacitor discharges. But, that doesn't happen instantaneously. Because, the rate of change of

Voltage of the capacitor is proportional to the current into it. That's a calculus concept. Or, more precisely, a differential equation. So, to fully understand the behavior of this circuit, you need calculus.

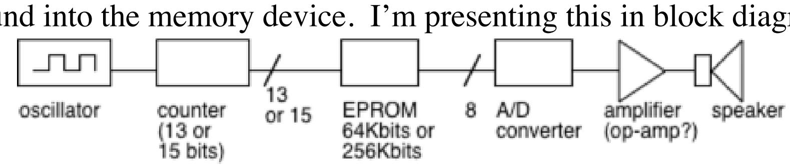
The problem as stated is a bit simpler. It's a "boundary value" problem that asks what is happening at the initial instant when the switch opens. At that point, the Capacitor is at 2 Volts. Using a "circuits" concept (Kirchoff's Voltage Law) we know that the sum of Voltages around a loop is zero. The switch has disconnected the second resistor from the circuit. So, applying Kirchoff's Voltage law, we find 4 Volts across the resistor. Using Ohm's Law, we can solve for resistor current to get 1mA, and that current is necessarily going into the capacitor due to Kirchoff's current law. A capacitor is characterized by $Q=CV$ and Q, Charge, is amperes times seconds. Differentiating thius (again, a calculus concept) gives $I = C\, dV/dt$ where C is capacitance in Ampere-seconds per Volt (called Farads), and dV/dt means the rate of change of Voltage. Solving, we have $dV/dt = I/C$ which gives us 10 Volts per second.

Pretty obviously, the capacitor will eventually get up to 6 Volts, but will approach that Voltage exponentially, with the rate of change decreasing gradually as the Voltage gets closer and closer to 6 Volts. This "exponential" response is characteristic of many, many engineering problems. In this case, it is a "first order" system because there is only one energy storing component. It is also a "linear" system because all of the components are characterized by proportionality relationships. Many practical engineering problems are like this, but much, much more complicated.

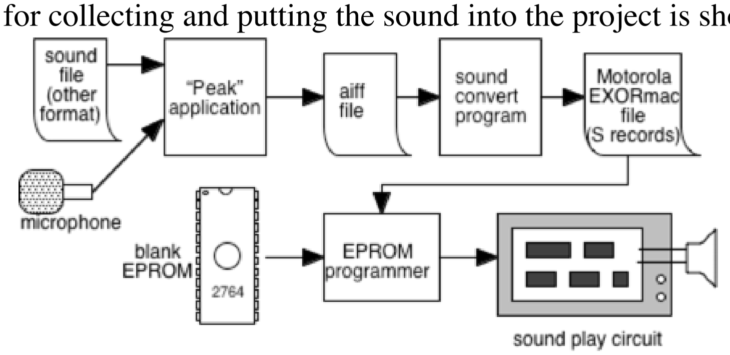
So, here's an example of a problem that draws heavily from several theories about components and circuits, uses calculus to mathematically characterize the system and solve for the one solution. Engineers solve these kinds of problems (but more complex) quite often.

8. How can your sound project circuit be improved? (The "sound project" was a digital circuit using a counter and a Read Only Memory, a D/A converter and an amplifier that could, when a button was pushed, produce an "interesting" sound. You could read this question as, "design a project that does this.")

To address this question, let me provide one solution to a "sound project" that was used in FYF101. This was the circuit the students built. (The sounds were recorded and each group put their own sound into the memory device. I'm presenting this in block diagram form:



The process for collecting and putting the sound into the project is shown below:



This problem is to improve the circuit. Or, is it to improve the whole project? What does "improve" mean? You see, before we can even begin talking about the improvements, or even the project itself, we need to understand what is sought. Cheaper? Simpler? More easily built? Bigger (more sound)? Higher fidelity? Louder? There are lots of possible ways to read the question, and each of those ways of understanding the question may have multiple answers.

I can only touch on a few of the possible answers. If we want "more sound" then we can make the memory bigger. In fact, when this was done, we actually used 512K bit EPROM (erasable memory) devices. At 8K samples of 8 bits per second (telephone sampling rate), that gives 8 seconds of sound. You want higher fidelity? The AIFF files were recorded at 16 bits, but the EPROM only stores 8 bits per sample. But then you need a good quality Digital to Analog converter, much better than the crude one we used, made out of resistors. And a better amplifier and speaker, too. You could use two EPROMs to get greater precision for each sample. You could go faster (40KHz, say) to get high fidelity sampling rates about equivalent to a modern high fidelity digital source. That means you store only 1.6 seconds of sound, though. Or, you could add complexity by storing the sound in a "compressed" format.

So, how do you decide which of these possible improvements to make? "It depends," is the answer. What are you going to use it for? What is the bound on what it will cost? Cost includes parts, labor, marketing, maintenance, maybe even end-of-life disposal. Who is the client? Is the client the user, or someone manufacturing a product for someone else? There are many, many questions to ask, and answer, before you can even start on the basic problem. This is the world many, maybe most, engineers inhabit. Problems don't exist in isolation. They are embedded in a world populated by people, companies, and governments, and must respond to the complexities of that environment. It's a complicated world that we live in.

9. What should be done to solve the problem of poverty?

This is what you call a "big problem." How do you even define it? What exactly does "poverty" mean? Governments try to define it. What exactly is, "The problem of poverty?" Is "poverty" a problem? For most of the history of civilization, it hasn't been, at least as far as those in positions of authority were concerned. Why is it a problem now, and what would a solution look like? What is being sought?

Just defining "poverty" is a problem. Someone who has no shelter, and cannot obtain enough food to sustain life, would be someone you'd consider "in poverty." But where is the boundary point between those in poverty and those not? Once, I purchased medicine for someone who had begged for my assistance, and delivered it to her home. She and her family had a large console TV in each of three main rooms, and obviously had a cable connection. She was receiving welfare for being "in poverty," but clearly was not in the "no shelter, insufficient food" state. How do you, or more importantly a government, define poverty?

Even more problematic, what can be done about it. Does giving money to individuals sufficient to close an income gap solve the problem? Well, not if that money is stolen, spent on drugs or drink, or gambled away. Should governments take control of people's lives to prevent such bad outcomes? That would amount to imprisonment.

This question is an example of a question so hard that it defies definition. People that discuss it have difficulty communicating because their concepts of what is being discussed often don't match. Without clear definitions communication is very difficult. Without definitions and an ability to communicate, solving a problem, if a solution can be found at all, becomes extremely difficult. This problem is very likely in the "unsolvable" category. That doesn't mean that attempts shouldn't be made to address it. It means that such attempts will result in frustration if the depth and difficulty of this kind of problem isn't appreciated.

The Point:

So, what's the point of all this? Maybe in school you've become used to every problem having one and only one answer. Or, worse, every problem having an answer that can be memorized, or worked according to a formula. It's time to get ready for a much more complicated world. We are trying to prepare you for that. Practice using your imagination.