

EE283 Laboratory Exercise #4 (Revised)

AC Components

Objectives:

1. To understand the controls and operation of a signal generator and oscilloscope
2. To understand the phase relationship between Voltage and Current in AC components:
Resistors, Capacitors, and Inductors
3. To understand the relationship of resistance / reactance to frequency

Theory:

4.0.1. Resistor:

Let the time varying Voltage across a resistor R , as shown in Figure 4.1(a), be

$$v(t) = V_M \cos \omega t \quad (4.1)$$

Where ω is the angular frequency (radians per second), $2\pi f$, frequency f is the number of cycles per second (units of Hertz), and t is time (in seconds).

From Ohm's Law, the current $i(t) = v(t)/R = V_M \cos \omega t / R$ (4.2)

Then, $i(t) = I_M \cos \omega t$ where $I_M = V_M / R$ (4.3)

Therefore, $i(t)$ and $v(t)$ are in phase, as shown in Figure 4.1(b).

In phasor form, $\mathbf{I} = \mathbf{V} / R$ (4.4)

Phasors \mathbf{V} and \mathbf{I} are in phase, as shown in Figure 4.1(c). (Note \mathbf{V} , \mathbf{I} for phasors in bold.)

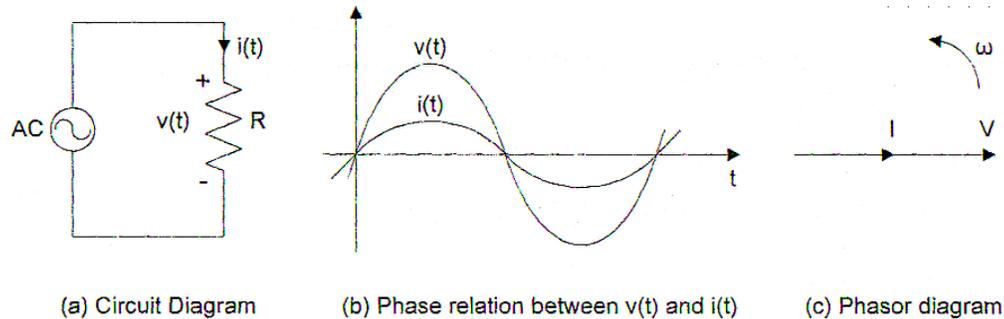


Figure 4.1 V-I Phase relations in a resistor

4.0.2. Capacitor:

Let the time varying Voltage across a capacitor C , as shown in Figure 4.2(a), be

$$v(t) = V_M \cos \omega t \quad (4.5)$$

Then, $i(t) = C dv(t)/dt = -\omega C V_M \sin \omega t = \omega C V_M \cos (\omega t + 90^\circ)$ (4.6)

So $i(t) = I_M \cos(\omega t + 90^\circ)$ where $I_M = \omega C V_M = V_M / X_C$ (4.7)

$X_C = 1/(\omega C)$ is called the capacitive reactance, and is inversely proportional to frequency.

Then $i(t)$ LEADS $v(t)$ by 90° as shown in Figure 4.2(b).

In Phasor form, $V = I X_C$ (4.8)

Where $X_C = -j/(\omega C) = 1/(j\omega C)$ (X_C will have units of Ohms after unit cancellations.) (4.9)

Then, I leads V by 90° , as shown in Figure 4.2(c).

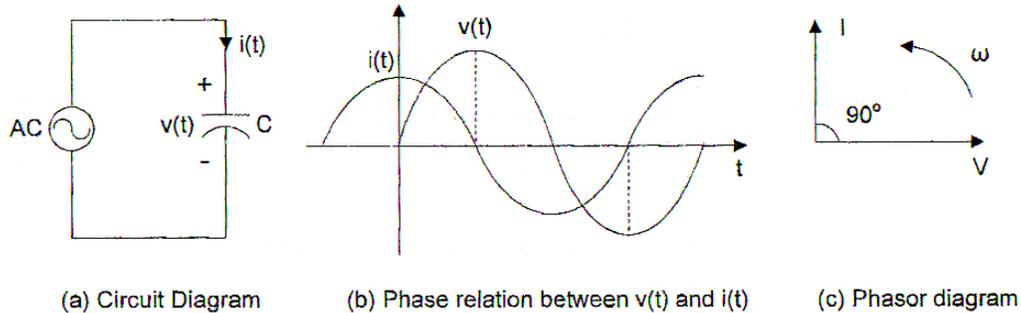


Figure 4.2 V-I Phase relations in a capacitor

4.0.3. Inductor:

Let the time varying Voltage across an inductor L , as shown in Figure 4.3(a), be

$$v(t) = V_M \cos \omega t \quad (4.10)$$

$$\text{Then, } i(t) = (1/L) \int v(t) dt = V_M \sin \omega t /(\omega L) = V_M \cos (\omega t - 90^\circ) /(\omega L) \quad (4.11)$$

$$\text{So } i(t) = I_M \cos(\omega t - 90^\circ) \text{ where } I_M = V_M/(\omega L) = V_M / X_L \quad (4.12)$$

$X_L = \omega L$ is called the inductive reactance, and is directly proportional to frequency.

Current $i(t)$ LAGS the Voltage $v(t)$ by 90° as shown in Figure 4.3(b).

In Phasor form, $V = I X_L$ (4.13)

Where $X_L = j\omega L$ (X_L will have units of Ohms after unit cancellations.) (4.14)

Then, I lags V by 90° , as shown in Figure 4.3(c).

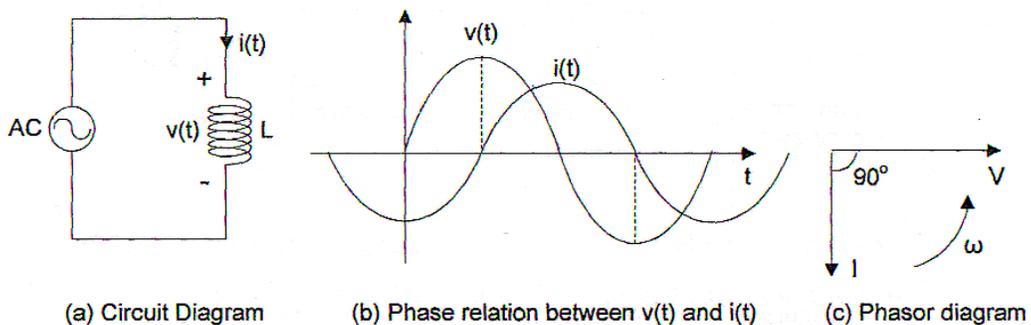


Figure 4.3 V-I Phase relations in an inductor

4.0.4. Root-mean-square (RMS) value

The root-mean-square value of an alternating Voltage or Current with period T is defined as:

$$X_{\text{RMS}} = \sqrt{\frac{1}{T} \int_0^T x^2(t) dt} \quad (4.15)$$

If $x(t) = X_{\text{max}} \cos(\omega t + \theta)$, $X_{\text{RMS}} = 0.707 X_{\text{max}}$ (applies to zero mean sinusoids) Note that in equation 4.15, “X” stands for either Voltage or Current, not Admittance or Impedance.

The purpose of an “RMS” measurement is to give a Voltage directly comparable to DC Voltage (or current) measurements. What AC Voltage dissipates the same power in a given resistor as 120 Volts DC? Household power is “120 Volts,” but it is actually 170 VAC peak and 340 VAC peak-to-peak. If you average the instantaneous power (V^2/R) delivered to the load over a full cycle (using integration) you get the RMS value of equation 4.15 above.

Procedure:

4.1 Basic set-up, oscilloscope, and function generator:

What we need to do is set up a loop circuit where we can observe and measure the AC Voltage across and current through a given component (the “unit under test”) Figure 4.4 Illustrates what we need. (This will apply for all three devices.) The current is measured by measuring the current across a small resistor and doing Ohm’s law: $i(t) = v(t)/R$.

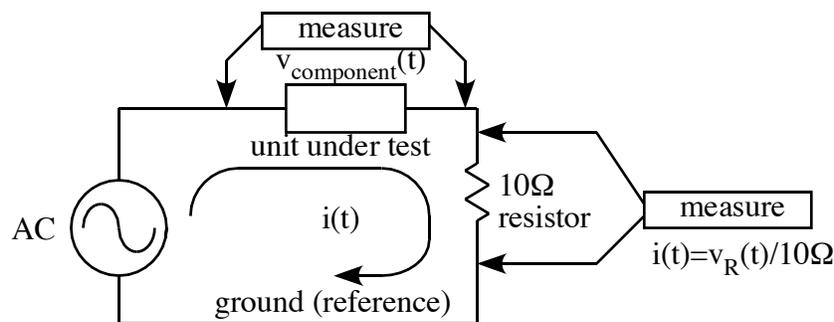


Figure 4.4 Determination of Voltage and Current for a component being tested (ideal case)

There is a problem: the signal generator and the digital oscilloscope grounds will be connected in common. As shown above, the Voltage measurement across the component has neither probe in common with both a connection to the signal generator and the other measurement channel. (The older analog oscilloscopes provided for “differential” channel inputs, so that both terminals of a channel could “float”. The modern digital oscilloscopes do not provide differential inputs (except using special, expensive probes).

What we will actually measure will be the Voltage of the unit under test, $v_1(t) = v_{\text{component}}(t)$, and MINUS the resistor Voltage $v_2(t) = v_{R(t)} = -i_{\text{component}}(t)$. See Figure 4.5 below. We can do this because the outer conductor of the signal generator is NOT actually grounded through the instrument as earth ground, as are the outer conductors of the oscilloscope. So, we can construct a circuit in which neither signal generator lead is grounded. Instead, we put ground (the shielded lead of each oscilloscope channel) at the node in between the unit under test and the resistor. The only disadvantage is that in the circuit current $i(t)$, when positive (in the direction of the arrow), will cause a negative Voltage to be indicated on Channel 2 of the oscilloscope. On the Channel menu, an option to “invert” the channel fixes that. Note: Do NOT use the T connector and BNC-BNC cable from the signal generator to Channel 1!

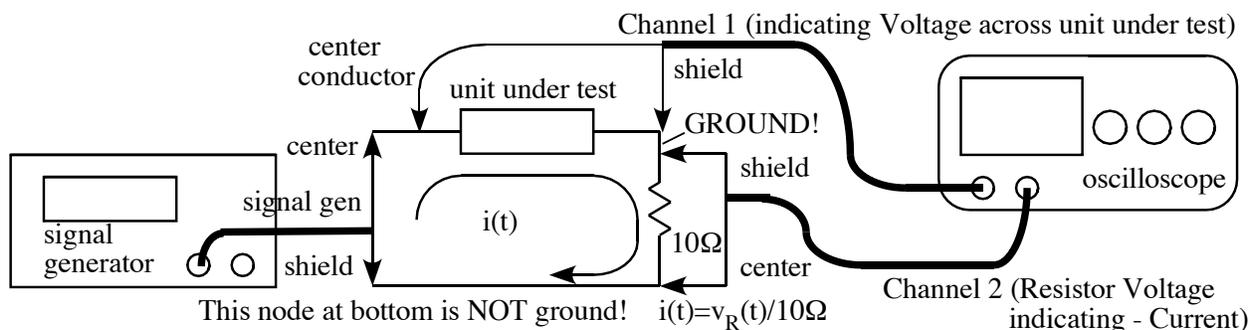


Figure 4.5 Test Circuit for Voltage and Current measurement

Refer to handouts on the oscilloscope and function generator. Make the following settings:

Set Oscilloscope to X versus T mode, time base to 1 msec per division, both channels to DC coupling, and 5 Volts per division (to start with). On the channel menu for Channel 2, make “invert” ON. Make sure both channels are set to “X1” for probes.

Set the signal generator, for Channel 1, to output a sine wave, at an amplitude of 10 Volts, no offset. Be sure to turn on the channel output.

4.2 Relation Between Voltage and Current for a Resistor

1. Measure the resistance of the 10 Ohm resistor used to measure current with the lab DMM. Also, check the measured resistance of the resistor substitution box to confirm that it has the desired resistance.
2. Connect the circuit shown in Figure 4.5. Choose a resistor R, about 470 Ω (as directed), to be the “unit under test.” Let channel 1 of the oscilloscope be the resistor under test Voltage. That is done with a BNC cable from Oscilloscope Channel 1 directly (NOT connected to the

“T” connector on the signal generator). Channel 1 of the oscilloscope needs to be across the resistor under test with Ground toward the 10 Ohm resistor. Use Channel 2 to measure the 10 Ohm resistor Voltage. Put ground toward the unit under test, so that both oscilloscope grounds are connected together. Warning: Be careful which lead is which. If you get the shield and center conductor backwards, you will short out the channel! Connect the signal generator with the center lead going to the resistor under test, and the outer lead (which, in this circuit, is NOT grounded) going to the end of the 10 Ohm resistor away from the oscilloscope grounds.

3. Set the function generator to configure Channel 1. (There is a button that selects which channel you are setting.) Select a sine waveform. Set Amplitude to 10 Volts (that’s peak Voltage. Peak to peak Voltage should be 20 Volts.) Set the frequency to the initial frequency given for the lab exercise (about 1KHz). (The Offset should remain at 0 Volts.)

4. The default mode of the oscilloscope should be signal vs. time (YT). If that’s not its mode, set “Display” Mode to YT. Set Channel 1 of the oscilloscope to 5 Volts per division. Make sure it has “DC coupling” and “X1” probe setting. (Higher quality oscilloscope probes are typically “X10”. Set the time base to 1 msec per division (of 1 cm). You should see the sinusoid appear on the screen. If it is not a stable waveform, adjust the “trigger” Voltage (knob). You might need to go to the Trigger menu to select Channel 1 as the trigger source. (You may also need to push the Channel 1 button to get Channel 1 to appear on the screen.) You should be able to observe that the signal displayed matches a 10 Volt peak (20 Volts p-p) 1KHz (or other frequency) sinusoidal waveform. Now adjust the oscilloscope time base to display just two or three cycles of the waveform.

5. Set channel 2 of the oscilloscope to 1 Volt per division. It should also be DC coupled, with “X1” probe. On the channel menu, turn “invert” ON. You may need to push the Channel 2 button for the trace to appear. You might be able to distinguish a trace, but the amplitude will be low. Use the (vertical) position controls to make sure the two channels don’t overlap. (You can turn down the scaling of Channel 1.) Now, you can adjust the Channel 2 sensitivity until you see a significant trace, probably when you get to 10mV per division. (You may see some noise on the signal.)

6. To review, Channel 1 is the tested resistor Voltage, and Channel 2 is the current (as indicated by minus the Voltage across the resistor). Adjust the waveforms to get a pleasing

display. You should see at least one complete cycle for each waveform. Position the channels vertically so that “zero” (indicated by an arrow mark at left) is at one of the grid lines (for reference). Now, sketch the Voltage (across the component) and Current waveforms on your laboratory report. Annotate these sketches now (or later) to be properly annotated graphs, meaning labels, units, numbers, grid lines and all the information needed to fully understand what $v(t)$ and $i(t)$ are doing. These properly annotated graphs are NOT just a copy of what you see on the screen. Your job is to interpret the information, not to be a camera. For current, your annotated graph needs to show the value in terms of current through the circuit, not Voltage across the resistor. Plot both with same zero point on the vertical axis, as in Figure 4.6 later. (I will also accept one graph under the other with separate x axes (for time) as long as the time axes are calibrated to the same scale to allow time comparisons, to get phase, to be made.) See the discussion for finding current in “Calculations” below. (Don’t sketch noise. Rather, estimate the centroid trace within the noise.)

7. Now, change the oscilloscope mode to XY. This shows the current vs Voltage relationship. Again, give a properly annotated graph for Current vs Voltage (Not Voltage vs Voltage). This is called a Lissajous pattern. When doing this, you can set both channels to zero (Ground) to properly position the origin (with both signals grounded, it’s just a dot on the screen) at the intersection of the two center grid lines.

Calculations:

Phase relationships:

The Voltage across a 10 Ohm resistor represents the current through the circuit, and hence through the unit under test. From the waveforms recorded, find the phase relationship. One full cycle represents 360° . An example is shown in Figure 4.6, where 16 divisions correspond to 360° , and the phase difference is 45° (2 divisions). $I(t)$ lags $v(t)$ by 45° .

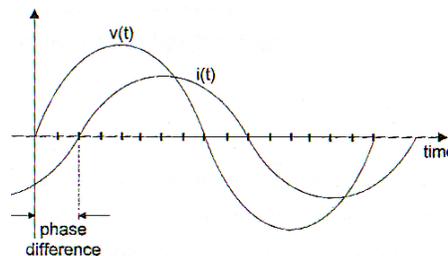


Figure 4.6 Example to illustrate the calculation of phase difference

Phase difference can also be calculated using the Lissajous pattern, as shown in Figure 4.7.

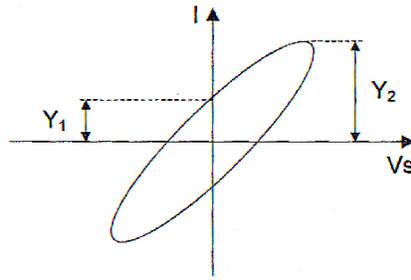


Figure 4.7 Lissajous pattern

If θ is the phase angle, then $\sin \theta = Y_1/Y_2$.

For the resistor, find the phase angle by both time domain traces and from the Lissajous pattern, and put those into your report. You should verify that, for a resistor, the current is in phase with the Voltage. With the Lissajous pattern displayed, change the frequency up and down an order of magnitude or so. Did the Lissajous figure change? If not, the current versus Voltage for a resistor is frequency invariant. (We will look at that again in Part 4.5.)

4.3 Relation Between Voltage and Current for a Capacitor

Replace the resistor (the unit under test, not the 10 Ohm current sensing resistor) with a 0.4 microFarad (or some other value as assigned) capacitor. Repeat steps 6 and 7, making any adjustments necessary, to generate traces for current and Voltage versus time for the capacitor, and then (similarly) a Lissajous pattern. Draw the waveforms on your report, again, making sure they are properly annotated. Be sure to indicate whether current LEADS or LAGS the Voltage. Calculate the phase relationship between current and Voltage both ways. After doing so, change the frequency up and down an order of magnitude, and notice how the Lissajous pattern changes. Do the changes make sense? What is changing, and why? (You can look at this as you change frequencies in the section below.)

4.4 Impedance vs Frequency Relationship for a Capacitor

Now, vary the frequency of the signal generator over a series of values as prescribed (between 250 Hz and 4KHz) and record the Voltage and the current measuring resistor Voltage as indicated on the oscilloscope at each frequency. Record these in a table (first three columns) using the peak-to-peak Voltages recorded from the oscilloscope. Later, convert Voltage to RMS, and calculate current as and RMS current. See “Calculations” below. The ratio of Voltage to Current (both RMS) is the “Impedance” of the capacitor.

(This is actually the absolute value of, or magnitude of, Impedance. The calculation can also be made with peak to peak or peak Voltages and currents, but both Voltage and current must be consistent.) The “Reactance” is the theoretical relationship between the Voltage and Current, which can be calculated given the capacitor value C and the frequency. The Impedance found from lab results should be very close to the Reactance found from the capacitor value and frequency.

4.5 Relation Between Voltage and Current in an Inductor

Replace the capacitor with an inductor with a specified inductance (about 70mH or so). Repeat steps 6 and 7 taken for the resistor, making any adjustments necessary to generate traces for current and Voltage versus time for the inductor. Then (similarly) produce a Lissajous pattern. Draw the waveforms on your report, again, making sure they are properly annotated graphs. Calculate the phase relationship between current and Voltage both ways. Be sure to indicate whether current LEADS or LAGS the Voltage. After doing so, change the frequency up and down, and notice how the Lissajous pattern changes. Do the changes make sense? What is changing, and why? (Do that as part of the following section.)

4.6 Impedance vs Frequency Relationship for an Inductor

Now, vary the frequency of the signal generator over a series of values as prescribed (between 250 Hz and 4KHz) and record the Voltage and the current measuring resistor Voltage as indicated on the oscilloscope at each frequency. Record frequency and these Voltages in a table using the peak-to-peak Voltages recorded from the oscilloscope (the first three columns). Later, convert Voltage to RMS, and calculate current as RMS current. See “Calculations” below. The ratio of Voltage to Current (both RMS) is the “Impedance” of the inductor. (It is actually the absolute value of, or magnitude of, Impedance. The calculation can also be made with peak to peak or peak Voltages and currents, but both Voltage and current must be consistent.) The “Reactance” is the theoretical relationship between the Voltage and Current, which can be calculated given the inductor value L and the frequency. The Impedance found from lab results should be very close to the Reactance found from the inductor value and frequency.

Calculations (for 4.4, 4.6):

After recording V_{p-p} across the resistor (used to measure current) and the Voltage (also p-p) across the unit under test, for each frequency, complete the tables by calculating the RMS Voltage and Current, from those values the impedance at each frequency. As seen earlier, because the current measuring resistor is 10 Ohms, $i(t) = v(t) / (10 \text{ Volts} / \text{Ampere})$. RMS values for Voltage and Current can be calculated from peak to peak values as shown in equation 4.15 earlier. In the last column of the table, calculate the Reactance for the unit under test, given the nominal value of the component and the frequency. These numbers should approximately match the results derived from your lab measurements. Do this for the capacitor and the inductor. (We are skipping the resistor since the reactance of a resistor does not change with frequency.)

The form report containing these tables and previous graphs and observations is due at the beginning of the next laboratory session.

4.7 Impedance vs Frequency graphs:

Using the tables of lab data for measured Impedance, plot on one graph the curves for the Capacitor and Inductor impedances versus frequency. Use a logarithmic scale for frequency and impedance. Also try plotting this data on reactance graph paper if available. Both graph data series are to be plotted against the same vertical and horizontal axes. This is to be done using the Excel program. Refer to the Engineering Laboratory Reports Manual for recommended techniques for plotting good graphs. The graphs are to be monochrome (black and white).

The graph, including an identification of the students, their lab section and station number, is to include nicely formatted tables (used to construct the graph) and the required graph. It should all be on one page. This is due at the beginning of the next lab session.

Comments:

This laboratory exercise has been revised to use the digital oscilloscopes rather than the analog oscilloscopes used earlier. The problem with not having differential inputs was discussed. The resistor frequency characteristics part of the earlier version of this exercise has been omitted.

EE283 Laboratory Exercise #4 AC Components Form report

Students: _____ **Section** _____ **Date:** _____
_____ **Station** _____

4.2 Relationship between Voltage and current for a Resistor R value = _____
10Ω resistor value: _____

(a) Voltage and current waveforms (b) Lissajous pattern
Phase from waveforms (show work): _____ Phase from Lissajous pattern (show work): _____

Any Observations and Comments (concerning the whole report):

4.3 Relationship between Voltage and current for a Capacitor Capacitor value: _____

(a) Voltage and current waveforms

(b) Lissajous pattern

Phase from waveforms (show work): _____ Phase from Lissajous pattern (show work): _____

4.4 Impedance vs Frequency relationships for capacitor

Table 4.4 Relationship between capacitance and frequency. C = _____ μF

| Frequency (Hz) | Voltage across C: V_C Volts(p-p) | Voltage across Rs: V_{R_s} Volts (p-p) | Voltage across C: V_C Volts(RMS) | Current through C ($I = V_{R_s} / R_s$) mA (RMS) | Impedance $ Z = V_C / I$ k Ω | Reactance $ X_C = 1/\omega C$ k Ω |
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4.5 Relationship between Voltage and current in an Inductor Inductor value: _____

(a) Voltage and current waveforms

(b) Lissajous pattern

Phase from waveforms (show work): _____ Phase from Lissajous pattern (show work): _____

4.6 Impedance vs Frequency relationship for Inductor

Table 4.6 Relationship between inductance and frequency. L = _____mH

| Frequency (Hz) | Voltage across L: V_L Volts(p-p) | Voltage across Rs: V_{Rs} Volts (p-p) | Voltage across L: V_L Volts(RMS) | Current through L ($I = V_{Rs} / R_s$) mA (RMS) | Impedance $ Z = V_L / I$ k Ω | Reactance $ X_L = \omega L$ k Ω |
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