EE283 Laboratory Exercise # 5

Frequency Selective Circuits (Filters)

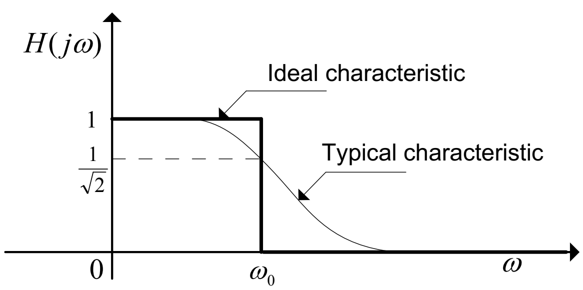
Objectives:

To understand:

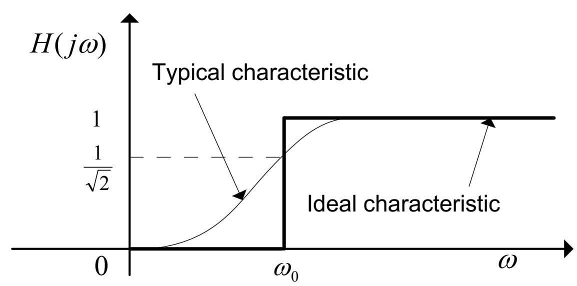
1. AC circuit analysis, and
2. Frequency selective circuits or filters

Theory:

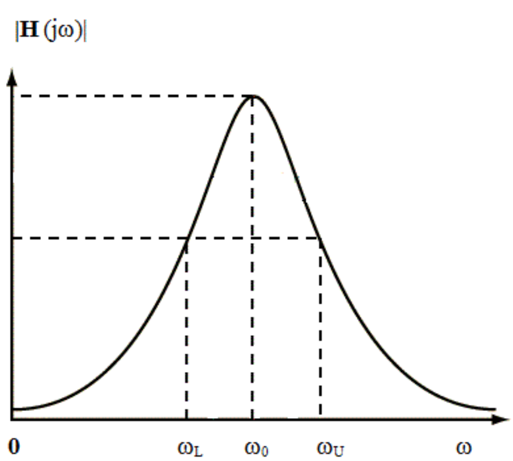
Frequency selective circuits, also known as filter circuits, allow signals within a band of frequencies called the passband, and reject or attenuate those within a band of frequencies called the stopband. Figure 5.1 shows the magnitude response, $|H(j\omega)|$, characteristics of various types of "ideal" filters. For low-pass and high-pass filters, ω_c is the cutoff or the break frequency. For band-pass and band-reject filters, ω_H and ω_L are the cutoff frequencies.



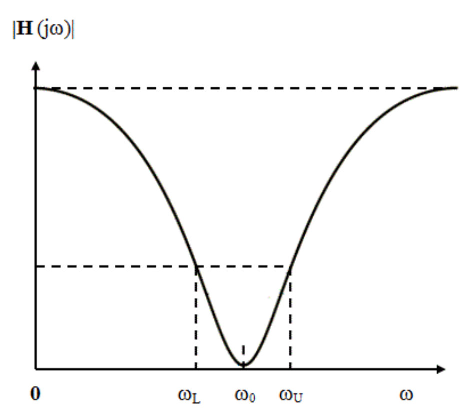
(a) Low-pass Filter



(b) High-pass Filter



(c) Band-pass Filter



(d) Band-reject Filter

Figure 5.1 Magnitude response characteristics of ideal filters

For band-pass and band reject filters, β is called the bandwidth, and is equal to ω_H - ω_L. The series RLC circuit to be investigated in this experiment is a band-pass filter.

A series resonant circuit shown in Figure 5.2. This particular circuit, thought of as a filter with output $\mathbf{V_O}$, would be a “band pass” filter. (Can you see why?) Expressions for resonant frequency and other quantities are derived next. (NOTE: Phasor quantities are shown in bold face.) (In practice, shunt circuits are usually used for bandpass circuits.)

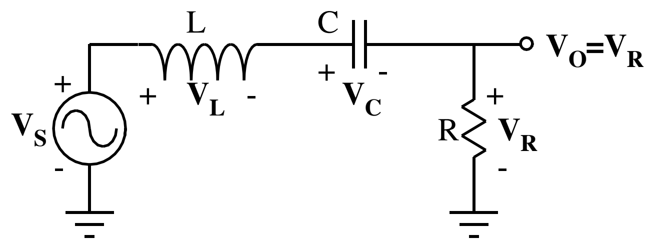


Figure 5.2 Series RLC circuit

Using KVL, $\mathbf{V_S} = \mathbf{V_L} + \mathbf{V_C} + \mathbf{V_R}$ (5.1)

where $\mathbf{V_S}$ = supply voltage phasor

$\mathbf{V_L} = \mathbf{IX_L}$ inductor voltage drop phasor (5.2)

$\mathbf{V_C} = \mathbf{IX_C}$ capacitor voltage drop phasor (5.3)

$\mathbf{V_R} = \mathbf{I}R$ resistor voltage drop phasor (5.4)

$\mathbf{I}$ = current phasor (5.5)

$\mathbf{X_L} = j\omega L = j2\pi fL$, the inductive reactance (5.6)

and $\mathbf{X}_c = \frac{1}{j\omega C} = \frac{-j}{\omega C} = \frac{-j}{2\pi fC}$, the capacitive reactance (5.7)

From equations (5.1) through (5.7)

$$\mathbf{V}_s = \mathbf{I}(R + j\omega L - \frac{j}{\omega C}) = \mathbf{IZ} \quad (5.8)$$

Then, the complex impedance of the circuit, **Z**, is

$$\mathbf{Z} = R + j(\omega L - \frac{1}{\omega C}) \quad (5.9)$$

At a particular frequency, when $j\omega L = \frac{1}{j\omega C}$ or equivalently, $j2\pi fL = \frac{1}{j2\pi fC}$

$$f = \frac{1}{2\pi\sqrt{LC}} = f_0 \text{ Hz} \quad (5.10)$$

This frequency f_0 is called the **resonant frequency**.

At resonance, the impedance **Z** = R, and is at a minimum.

At frequency f_0, the current $|I| = \frac{V_s}{R}$ and reaches a maximum value for a given voltage V_s.

Phasor Diagram at Resonance:

The resonance condition at f_0 shown using phasors is illustrated in Figure 5.3.

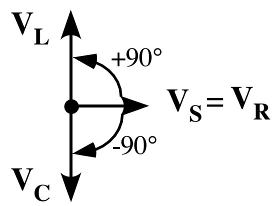


Figure 5.3 Phasor diagram at resonance

As can be seen, at resonance, current is maximum, power factor is unity, impedance is equal to resistance, $|V_L| = |V_C|$, and each is equal to the quality factor times $|\mathbf{V}_s|$.

At lower frequencies, the circuit is capacitive, since $\mathbf{X_C} > \mathbf{X_L}$, and at higher frequencies, it is more inductive since $\mathbf{X_L} > \mathbf{X_C}$.

Half-power Frequencies: Frequencies f_1 and f_2 at which the current is $\frac{1}{\sqrt{2}}$ times the maximum current $\frac{V_s}{R}$, or equivalently, the power lost in R is one-half of the power lost at resonance, are called the half-power frequencies. They can be calculated as follows:

$$f_1 = \frac{1}{2\pi}\left\{\left(-\frac{R}{2L}\right) + \sqrt{\left[\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}\right]}\right\} \text{ Hz} \qquad (5.11)$$

$$f_2 = \frac{1}{2\pi}\left\{\left(+\frac{R}{2L}\right) + \sqrt{\left[\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}\right]}\right\} \text{ Hz} \qquad (5.12)$$

Bandwidth: It is defined as

$$\beta = (f_2 - f_1) \text{ Hz} \qquad (5.13)$$

$$= \frac{R}{2\pi L} \text{ Hz} \qquad (5.14)$$

and represents the band of frequencies between f_1 and f_2 in which the current is high.

Quality Factor: It is defined as the ratio of the resonant frequency to the bandwidth.

$$Q = \frac{f_0}{\beta} \qquad (5.15)$$

Alternate expressions of Q are:

$$Q = \left(\frac{1}{R}\right)\sqrt{\frac{L}{C}} = \frac{2\pi f_0 L}{R} = \frac{1}{2\pi f_0 CR} \tag{5.16}$$

At resonance, $|V_L| = |V_C| = QV_s$ (5.17)

i.e., the voltage across L and that across C are each equal to Q times the supply voltage.

Procedure:

1. Choose the values of R, L, and C according to the specifications provided by your instructor. (Example: Desired resonant frequency is 5 kHz; quality factor is 10. R is about 500 Ω.)
2. Calculate the values of f_0, f_1, f_2, β, and Q using equations (6.10) through (6.15)
3. Construct the circuit as shown in Figure 6.4. Use the DMM to measure V_L and V_C. (Use the same DMM to measure each voltage. You can move one lead back and forth, while leaving the other lead connected to the node between the inductor and the capacitor. Since these are AC measurements, polarity doesn't matter.) V_S and V_R are monitored by the vertical channels 1 and 2 of the oscilloscope, respectively. Readings at frequencies that give nothing but noise waveforms are to be omitted. (Zeros would be correct, but are a problem for logarithmic plots later.) Refer to Lab #4 and tutorial materials for the procedures for setting up the oscilloscope and the function generator.

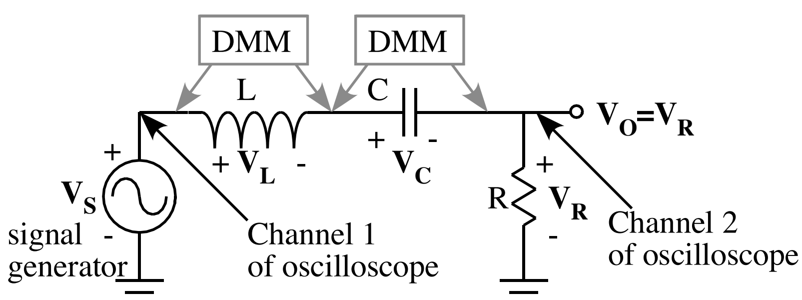


Figure 6.4 Circuit connection to determine the resonant frequency

4. Prepare the function generator to supply a sinusoidal signal with zero DC offset. Set the supply voltage be about 4V (p-p).
5. You are required to take at least 5 sets of readings at frequencies below the calculated resonant frequency, one at or close to the resonant frequency, and at least 5 sets above the resonant frequency. It is preferable that these frequencies are nearly uniformly distributed. Set the frequency at the lowest value chosen, and record the values of frequency f, Voltage magnitudes for $\mathbf{V_S}$, $\mathbf{V_R}$, $\mathbf{V_L}$, and $\mathbf{V_C}$, and the phase angle difference θ between $\mathbf{V_S}$ and $\mathbf{V_R}$.

Since the current **I** and voltage $\mathbf{V_R}$ are in phase, the angle θ is also the phase angle between $\mathbf{V_S}$ and current **I**. Mark the phase angle θ positive if $\mathbf{V_R}$ leads $\mathbf{V_S}$ and negative if it lags.

6. Repeat the above step for all other frequencies.
7. Calculate the current I as $\frac{V_s}{R}$. Calculate the power factor as cos θ and specify whether it is lagging (θ is negative), leading (θ is positive) or in phase.
8. Produce a neat graph of $\left|\frac{V_L}{V_S}\right|$ and $\left|\frac{V_C}{V_S}\right|$ versus frequency f with f on the common x-axis.

   Use a semi-log plot with frequency on log scale. Use the Excel program. Notice that V_S does not actually remain constant, even though you do not change the signal generator setting. That is because the internal resistance of the signal generator (75Ω) becomes significant at the resonant frequency.

   From the graph, decide the values of the resonant frequency f_0 (frequency at which the current is maximum), the half-power frequencies f_1 and f_2 (frequencies at which the current is 0.707 times the maximum current), and the bandwidth β, and the quality factor Q.
9. Enter all the calculated and measured values of f_0, f_1, f_2, β, and Q in a table. (Why might the observed and calculated values be different, if they are?)
10. Annotate the graph to show the f_L, f_0, and f_H. Print it, and include it in your report.

Report:

Print the graph and append it to the laboratory report, which is handed in at the end of the lab period with the form report containing the tables. (A properly formatted Excel table is an acceptable substitute for the hand-written table on the report form. If you do that, make sure all the information designated on the form report is in the report that you hand in.) Be sure to convert oscilloscope readings for Voltage into RMS values (from peak or peak-to-peak, whichever you read).

Comment: One big change from previous years is that students are expected to use their computer, in the lab, to enter data into a table, and produce a graph, which is included with the form report at the end of the lab exercise. To do this well, they need to have made a point of mastering the use of Excel ahead of time. (We also need to make darned sure the printer is working and has enough paper!)

EE283 Laboratory Exercise # 5

Frequency Selective Circuits (Filters)

Specifications: Resistance R =_______; Inductance L = ______ ; Capacitance C =__________

Nominal Supply Voltage setting for V_S =_______________

Table 5.1 Readings of Series Resonance Circuit

Frequency kHz	Supply Voltage V_S (rms)	Inductor Voltage V_L Volts(rms)	Capacitor Voltage V_C Volts(rms)	Resistor Voltage V_R Volts(rms)	Current I mA	power factor angle θ degrees	power factor lead/lag	ratio $\frac{V_L}{V_S}$	ratio $\frac{V_C}{V_S}$

Table 5.2 Comparison of Observed and Calculated Values

	resonant frequency f_o kHz	lower half-power frequency f_1 kHz	upper half-power frequency f_2 kHz	Bandwidth β kHz	Quality factor Q
calculated					
observed					

Append properly graph as described in the instructions.