

EE283 Laboratory Exercise #6 AC Circuit Analysis

Objective:

To understand the concept of node analysis and the use of phasors for AC circuits

Theory:

1. Kirchhoff's Voltage Law and the Voltage Division Principle:

Kirchhoff's Voltage Law (KVL) states that the algebraic sum of the voltages across elements around any closed path (loop) in an AC circuit is zero, just as for DC circuits. Figure 6.1 illustrates. V_s = the sum of all circuit series component Voltages.

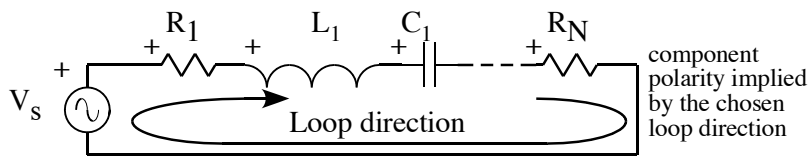


Figure 6.1 Series AC Circuit

According to the Voltage division principle, the Voltage drop across any component in an AC series circuit is proportional to the impedance of that component. The Voltage drop V_x across any component Z_x can be shown to be:

$$V_x = (V_s / Z_{eq}) Z_x \quad (6.2)$$

where the equivalent (total) impedance of the circuit is given by:

$$Z_{eq} = Z_1 + Z_2 + \dots + Z_N \quad (6.3)$$

2. Kirchhoff's Current Law and the Current Division Principle:

Kirchhoff's Current Law (KCL) states that the algebraic sum of the currents at any node in an electric circuit is zero.

See Figure 6.2. Using the convention that the current entering a node is negative, and that leaving the node is positive, the KCL equation is given below. (Note: Currents are phasors.)

$$-I_s + I_1 + I_2 = 0A \quad (6.4)$$

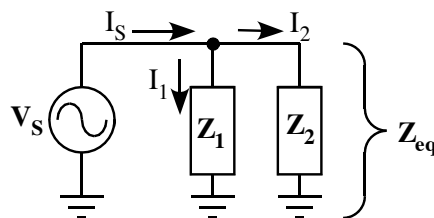


Figure 6.2 Currents at a node of an AC circuit

According to current division principle, the current through an impedance in an AC parallel circuit is inversely proportional to its impedance value.

The current I_X through impedance Z_X can be shown to be

$$I_X = (I_S Z_{eq} / Z_X) \quad (6.5)$$

where Z_{eq} is the equivalent impedance of the parallel impedances Z_1 and Z_2 . The general case equation is:

$$(1/Z_{eq}) = (1/Z_1) + (1/Z_2) + (1/Z_3) \dots + (1/Z_N) \quad (6.6)$$

In this case, there are only two impedances in parallel, so $(1/Z_{eq}) = (1/Z_1) + (1/Z_1)$.

This can be algebraically simplified to: $Z_{eq} = (Z_1 Z_2) / (Z_1 + Z_2)$.

3. Node Analysis:

In a circuit with N nodes, one node is chosen to be the reference node (“ground”) and an arbitrary Voltage (usually zero) assigned to it. If the voltages at the remaining (N-1) nodes are unknown, then KCL equations can be written at these (N-1) nodes, and the resulting (N-1) simultaneous equations can be solved for (N-1) unknown node Voltages. Any branch current can then be calculated using $V=Z I$. In Figure 6.3, there are three nodes other than ground.

However, in this case, the V_S node Voltage is known. So, we get $N = 2$.

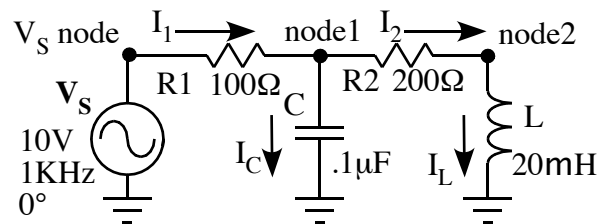


Figure 6.3 Circuit to explain AC node analysis

Choosing the reference (ground) node, and skipping the known node at the Voltage source, KCL equations for the arbitrary (left to right, top to bottom) current directions shown are:

$$\text{Node 1: } - \frac{(10V \angle 0^\circ - V_{n1})}{100\Omega} + \frac{(V_{n1})}{1/(2K\pi j(.1\mu F))} + \frac{(V_{n1} - V_{n2})}{200\Omega} = 0V \quad (6.7)$$

$$\text{Node 2: } - \frac{(V_{n1} - V_{n2})}{200\Omega} + \frac{(V_{n2})}{2K\pi j(20mH)} = 0V \quad (6.8)$$

The convention used here is that a current is taken as positive when it is leaving a node and negative when it enters a node. Solving the equations simultaneously is difficult, since complex algebra is required. The problem is most easily solved in matrix form using MATLAB.

$$Z_C = 1/(j\omega C) = 1.592K\Omega \angle -90^\circ$$

$$Z_L = j\omega L = 126\Omega \angle 90^\circ$$

Solving at node 2 (Equation 6.8): $-V_{n1}/200\Omega + V_{n2}/200\Omega + V_{n2}/(126\Omega \angle 90^\circ) = 0V$

$$\text{Gets us: } V_{n1} = (1 - 1.59j) V_{n2}$$

Substituting into the node 1 equation (6.7):

$$\frac{-(10V - (1-1.59j) V_{n2})}{100\Omega} + \frac{(1-1.59j) V_{n2}}{1.592K\Omega \angle -90^\circ} + \frac{(1-1.59j-1) V_{n2}}{200\Omega} = 0V$$

$$\text{This reduces to: } (2(1-1.59j) + .1256j(1-1.59j) - 1.59j) V_{n2} = 20V$$

$$\text{And we solve for: } V_{n2} = 3.89V \angle 64.7^\circ$$

$$\text{Back substituting for } V_{n1} = (1 - 1.59j) V_{n2} = 7.31 \angle 6.9^\circ$$

Branch currents can then be calculated using $V=ZI$. In this example, to take two cases:

$$I_1 = 2.88V \angle -17.5^\circ / 100\Omega = 28.8mA \angle -17.5^\circ \quad I_C = 7.31V \angle 6.9^\circ / 1.592K\Omega \angle -90^\circ = 4.59mA \angle 96.8^\circ$$

(See Appendix for MATLAB and Excel Solutions)

Procedure:

1. Choose components as assigned by the instructor. These will include an assortment of resistors, capacitors, and an inductor. Note the assigned circuit given for your section, and build the circuit. Connect it to the signal generator, which will serve as the AC source.

Measure all the node Voltages and the supply Voltage(s) using the oscilloscope. Be sure to include phase angle, in reference to the supply which is considered to be at 0° .

Record node Voltages on the form provided. Turn in the form at the end of the lab period.

2. Make calculations as follows.

a. Calculate the Node Voltage phasors based on a Node Analysis of the given circuit. (Use of MATLAB or Python is strongly encouraged.) Calculate branch currents. Show your work.

b. Calculate the branch currents based on node Voltages observed in the laboratory, for each component. Verify against the calculated branch currents from part 1 to see whether KCL is verified at each node. (If not, your measurements and/or calculations are wrong. You need to figure out what happened.)

3. Use computer simulation (PSpice or LTSpice) to verify your results.

Reports:

Turn in the form report for this Laboratory Exercise. This form will have the results and measurements you obtained in the laboratory.

At the beginning of the following laboratory exercise, hand in the written report for this lab exercise, which includes the circuit analysis and simulation materials associated with parts 2

and 3. The written report must include a cover page giving a title, the students' names, section, and lab station. The node analysis needs to include the figure showing the circuit, a table of the lab results, the calculated results from node and loop analysis, and simulation results. These need to be presented in a manner to make a comparison easy. Append the simulation schematic and results image for the node analysis circuit. It should look something like Figures 6.4 and 6.5

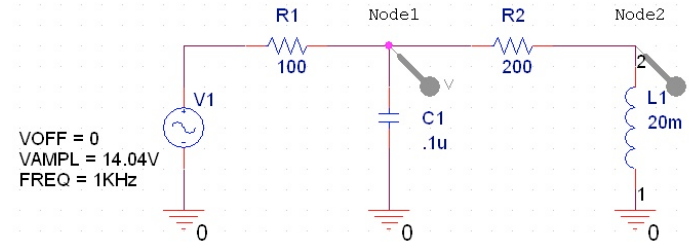


Figure 6.4 Example of AC Circuit Simulation Circuit

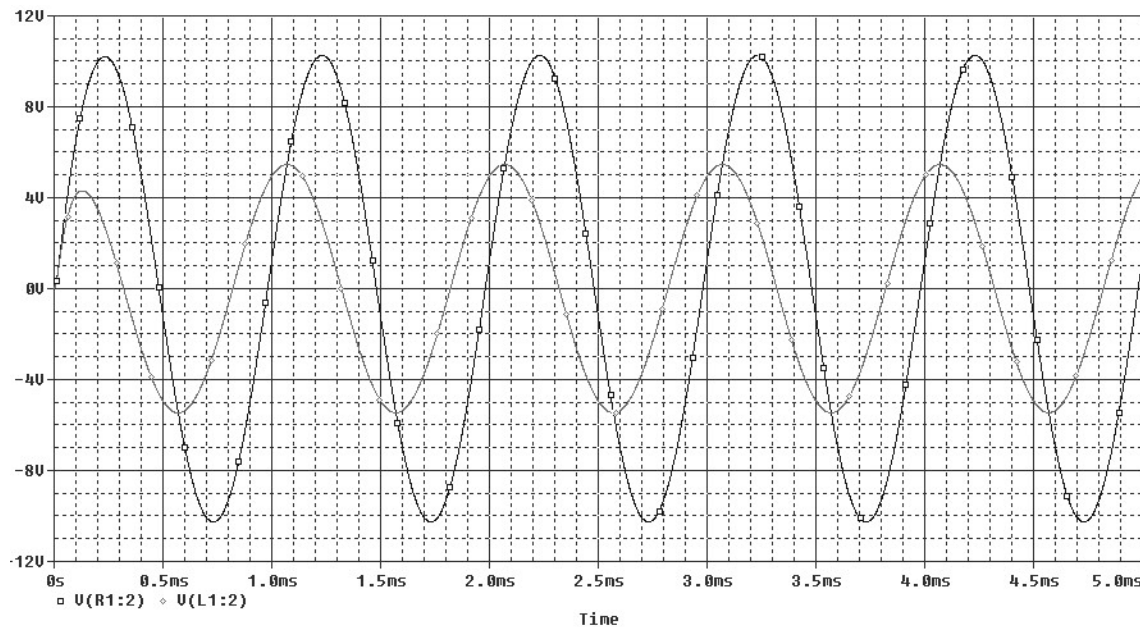


Figure 6.5 Example of AC Circuit Simulation Results (Node Voltages vs Time)

The report due with the Node analysis and simulation results is to also to include the following:

1. Table 6.1 should give, for each node: Node Voltage from Node Analysis, Node Voltage from lab results, and Node Voltage from simulation. These are phasors referenced to V_s .
2. Give a graph showing Lab node waveforms.
3. Table 6.2 should give, for each component: Calculated Voltage, by Node Analysis, Calculated Voltage from Lab results, and Voltage determined from the simulation. (These are phasors.)

4. Your work showing how you did the node analysis, including your MATLAB, Python, or Excel code and results or the work done by whatever means you employed. If you did the entire solution pencil and paper (not encouraged) make sure your work is neat and includes enough remarks to be easily followed. It does not have to be typed.

A conclusions paragraph is to comment on how well the observations met expectations. Point out any significant discrepancies. Give the reason for discrepancies. Use analytic techniques to do so to the extent that you can. (Don't just guess.)

The report should be well organized and include, of course, administrative details like names, section, and lab station.

Appendix

EXCEL Spreadsheet used to solve the Example Problem:

This is the solution to the equation at the top of page 3 in EE283 Lab Exercise 6							
1-1.59i	2	2-3.18i					C3 is the first term
0+0.1256i		0.199704+0.1256i					C4 is the second term
		0-1.59i					C5 is the third term
		2.199704-4.6444i					C6 is the sum of the three terms
		1.66586511063564+3.51726592297699i					C7 is Vn2
		3.891820415					C8 is the magnitude of Vn2
		1.128470456					Angle of Vn2 in radians
		64.65659443					Angle of Vn2 in degrees
		7.25831792816905+0.868540397066322i					C12 is Vn1
		7.310098602					C13 is the magnitude of Vn1
		6.823647012					C14 is the angle of Vn1 in degrees
This is the solution for the I1 and Ic currents							
		0.0274168207183095-0.00868540397066322i					C18 is I1 in A
		0.028759664					C19 is the magnitude of I1 in A
		-17.57773671					C20 is the angle of I1 in degrees
		-0.000545565576046685+0.00455924492975443i					C22 is Ic in A
		0.00459177					C23 is the magnitude of Ic in A
		96.82364701					C24 is the angle of Ic in degrees

Appendix: MATLAB AC Circuit Node Analysis

```

% EE283 Assignment 6 AC Node Analysis
% JB Gilmer 8/15/17 script file EE283L6.m
% Inputs: Parameters of matrices A and b for node analysis (in file)
% Purpose: Find the Node Voltages and branch currents of circuit
% Outputs: Node Voltages V1 and V2, Currents I1, I2, IC, IL
format short g
format compact
% Definition of circuit parameters
f=1000; %Hz
R1=100; %Ohms
R2=200; %Ohms
ZC=1/(j*2*pi*f*.1e-6); %Ohms C=0.1uF
ZL=j*2*pi*f*20e-3; % Ohms L=20mH
VS=10.0; %Volts (at 0 degrees)
%Circuit
%      |---\\/\|/---V1---\\/\|/---V2
%      |      R1      |      R2      3
%      VS          = C          3 L
%      |              |          |
%      Gnd            Gnd        Gnd
%Equations
% EQ1 -(VS-V1)/R1 + V1/ZC + (V1-V2)/R2 = 0
% EQ2 -(V1-V2)/R2 + V2/ZL = 0
% A and b matrix parameters for Ax=b x=[V1 V2]'
a11= 1/R1+1/ZC+1/R2;
a12= -1/R2;
a21= -1/R2;
a22= 1/R2+1/ZL;
A=[a11 a12;a21 a22];
b=[10/R1 0]';
%Solve for V1 and V2
x=inv(A)*b;
V1 = x(1)
V2 = x(2)
%Find branch currents
I1 = (10-V1)/R1
I2 = (V1-V2)/R2
IC = V1/ZC
IL = V2/ZL
%Find node net currents
IN1= -I1+I2+IC
IN2= -I2+IL

```

Execution in MATLAB session:

```

>> EE283L6
V1      7.2575 + 0.86763i
V2 =    1.6633 + 3.5149i
I1 =    0.027425 - 0.0086763i
I2 =    0.027971 - 0.013236i
IC =   -0.00054515 + 0.00456i
IL =    0.027971 - 0.013236i
IN1 =   1.0083e-017 +8.6736e-019i (This is close to zero)
IN2 =          0 +1.7347e-018i (This is close to zero)

```

Appendix: Source Code in Python for EE283_lab6.py

```

# EE283 Assignment 6: AC Node Analysis
#
# Inputs:
# * source voltage, VS (in Volts)
# * source frequency, f (in Hz)
# * resistance, R1 (in Ohms)
# * resistance, R2 (in Ohms)
# * capacitance, C (in Farads)
# * inductance, L (in Henrys)
#
# Outputs:
# * node voltages (in Volts): V1, V2
# * node currents (in Amps): IN1, IN2
# * branch currents (in Amps: I1, I2, IC, IL
#
import numpy as np

circuit = """
    |---/\//\---V1---/\//\---V2
    |           |           |
    VS          R1          = C          R2          3 L
    |           |           |           |           |
    Gnd         Gnd        Gnd        Gnd        Gnd
    """

# Circuit Parameters
f = 1000 # Hz
VS = 10.0 # Volts
R1 = 100.0 # Ohms
R2 = 200.0 # Ohms
C = 0.1E-6 # Farads
L = 20.0E-3 # Henrys

# Impedances calculated from f, L, and C
ZC = 1 / (2j * np.pi * f * C) # Ohms
ZL = 2j * np.pi * f * L # Ohms

# Represent impedance and currents as matrix A and vector b
A = np.array([[1 / R1 + 1 / ZC + 1 / R2, -1 / R2],
              [-1 / R2, 1 / R2 + 1 / ZL]])
b = np.array([VS / R1, 0])
x = np.linalg.solve(A, b)

# Calculate voltages and currents
V1, V2 = x[0], x[1]
I1 = (VS - V1) / R1
I2 = (V1 - V2) / R2
IC = V1 / ZC
IL = V2 / ZL
IN1 = -I1 + I2 + IC
IN2 = -I2 + IL

# Print all results
print("Circuit")

```

```

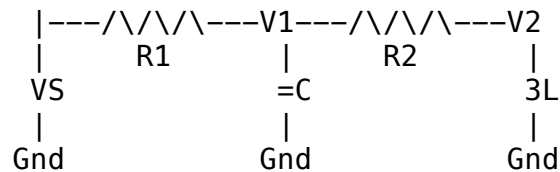
print("-----")
print(circuit)
print()
print("Parameters")
print("-----")
print("Source Frequency: f = %.4f" % f)
print("Source Voltage: VS = %.4f" % VS)
print("Resistors: R1 = %.4f, R2 = %.4f Ohms" % (R1, R2))
print("Impedances: ZC = %.4f %+.4fj, ZL = %.4f %+.4fj Ohms" %
      (ZC.real, ZC.imag, ZL.real, ZL.imag))
print()
print("Results")
print("-----")
print("V1 = %.4f %+.4fj, V2 = %.4f %+.4fj Volts" %
      (V1.real, V1.imag, V2.real, V2.imag))
print("I1 = %.4f %+.4fj, I2 = %.4f %+.4fj Volts" %
      (I1.real, I1.imag, I2.real, I2.imag))
print("IC = %.4f %+.4fj, IL = %.4f %+.4fj Volts" %
      (IC.real, IC.imag, IL.real, IL.imag))
print("IN1 = %.4f %+.4fj, IN2 = %.4f %+.4fj Volts" %
      (IN1.real, IN1.imag, IN2.real, IN2.imag))
print("*IN1 and IN2 should be close to zero*")

```

Output from running program:

```
> python EE283_lab6.py Circuit
```

```
-----
```



Parameters

```
-----
```

```

Source Frequency: f = 1000.0000
Source Voltage: VS = 10.0000
Resistors: R1 = 100.0000, R2 = 200.0000 Ohms
Impedances: ZC = 0.0000 -1591.5494j, ZL = 0.0000 +125.6637j Ohms

```

Results

```
-----
```

```

V1 = 7.2575 +0.8676j, V2 = 1.6633 +3.5149j Volts
I1 = 0.0274 -0.0087j, I2 = 0.0280 -0.0132j Volts
IC = -0.0005 +0.0046j, IL = 0.0280 -0.0132j Volts
IN1 = 0.0000 +0.0000j, IN2 = -0.0000 +0.0000j Volts
*IN1 and IN2 should be close to zero*

```

```
>
```