#### Experiment 9a: The No Load Cantilevered Beam Modeled as a Spring/Mass System— **Effective Mass Considerations** (Application: Piper Cub Airplane Wing)

# **Purpose:**

To model the no load cantilevered beam as a spring mass/system to obtain an equivalent effective mass of the beam in a spring mass context.

## **Questions:**

1.) The natural frequency of an undamped spring mass system is:

$$\omega_n = \sqrt{\frac{k_{effective}}{m_{effective}}} \tag{1}$$

Solving this for m yields:

$$m_{effective} = \frac{k_{effective}}{\omega_n^2}$$
The spring constant of the beam is given by:

$$k_{effective} = \frac{Ebh^3}{4L^3} \tag{3}$$

 $k_{effective} = \frac{{}^{Ebh^3}}{{}^{4L^3}}$  Experimentally, the natural frequency is given by:

$$\omega_n = \frac{2\pi}{T} \tag{4}$$

Substituting equations 3 and 4 into equation 1 yields:

$$m_{effective} = \frac{\frac{Ebh^3}{4L^3}}{\left(\frac{2\pi}{T}\right)^2} \tag{5}$$

Simplifying yields:

$$m_{effective} = \frac{T^2 Ebh^3}{\pi^2 L^3} \tag{6}$$

**Expected Slope Calculation:** 2.) .144 = .028 \* .0018 \* 2700

The expected slope is the linear density of the aluminum. This experiment has demonstrated that the effective linear density for the vibrating beam is much less than its actual density.

### Data:

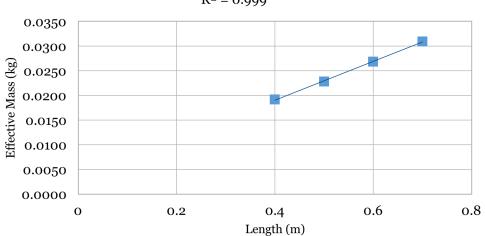
Table I: Results

Length (m)	ω <sub>n</sub>	Δ	Effective Mass (kg)	
			Equation 7	Equation 8
0.4	52.360	0.003578	0.0192	0.0192
0.5	34-334	0.008322	0.0228	0.0228
0.6	24.074	0.016927	0.0268	0.0268
0.7	17.799	0.030964	0.0309	0.0309

Table II: Slope and Linear Fit

(a)	Slope (kg/m)	Expected
Figure 1	0.039	0.144
Figure 2	0.039	0.144

y = 0.039x + 0.003 $R^2 = 0.999$ 



Figure

1: Effective Mass as a Function of Length for Equation 7

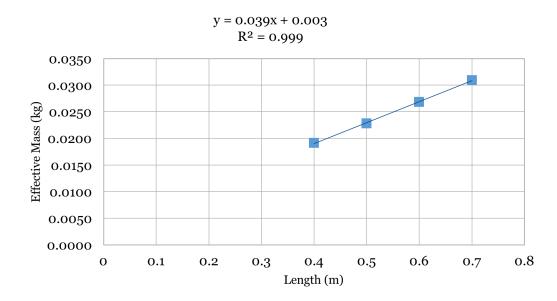


Figure 2: Effective Mass as a Function of Length for Equation 8

#### **Conclusions:**

This experiment has demonstrated how the lineal density of a vibrating beam is less than its actual lineal density. While this is a curious discovery it may not be valid. This lab required the static deflection to be computed from the natural frequency. This requirement means Equations 7 and 8 in the manual are coupled, explaining why identical results were calculated from each equation. The data may be in error due to this coupling.

#### **Comments:**

This seems more of an informal report than a formal report. It does not have an abstract. There is no description of the experimental apparatus or procedure. The results are presented in tabular form with no text and callouts. This is all perhaps acceptable in an informal report. I say "perhaps" because expectations for an informal report are much more open to interpretation and possible misunderstanding. Just what is an "Informal Report"? It could be anything from filling out a form to something just short of all the requirements of a formal report. If you are asked for an informal report, be sure you know exactly what the expectations are. In any case, even for an informal report, correct annotations and formatting of graphs and tables is still required, and you must use proper units for data. Generally, an informal report will require a conclusions section, and it may require an abstract (though, apparently, not in this case).

This document was converted from an ".odt" file (LibreOffice application for word processing) which is why the tables were converted from the original document to jpg files in order to be inserted into this Word document. The misalignment of the title for Figure 1 is also an artifact of conversion to a Word document from the original, which was correct.

Putting the goodness of fit equations on the graphs would normally not be done in a formal report; they would be in the text. For this informal report, with no text in the results section, the placement used is satisfactory.

This exercise is on a blurry line between an "experiment" and a "characterization". Calling it an "experiment" is appropriate because the performance is compared to what theory predicts. The implicit hypothesis is that theory accurately predicts the behavior. In this case, there is a significant difference between theory and experimental results. The reader wonders, "Why?" Some discussion is given to the issue, but no method for resolving the discrepancy is recommended.