EE283 Lab 6
AC RLC Node Analysis

Objective:

To understand the concept of node analysis and the use of phasors for AC circuits.

Theory:

1. **Kirchhoff's Voltage Law and the Voltage Division Principle:**

Kirchhoff's Voltage Law (KVL) states that the algebraic sum of the voltages across elements around any closed path (loop) in an AC circuit is zero, just as for DC circuits. Figure 1 illustrates this principle. In this figure V_S equals the sum of all circuit series component voltages.

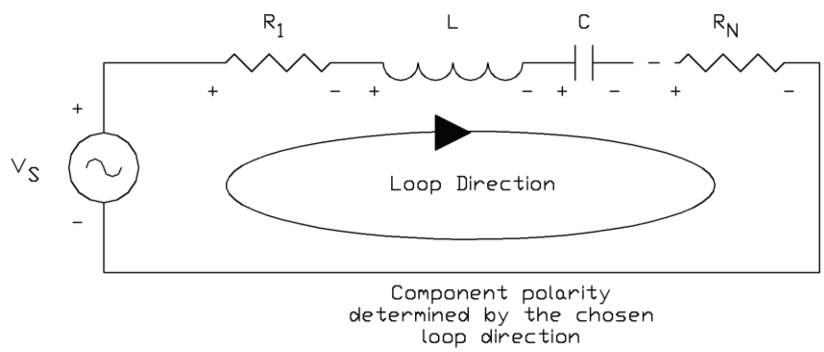


Figure 1
Series AC Circuit

According to the voltage division principle the voltage drop across any component in an AC series circuit is proportional to the impedance of that component. The voltage drop $\mathbf{V_X}$ across any component $\mathbf{Z_X}$ can be shown to be:

$$\mathbf{V_X} = \left(\frac{\mathbf{V_S}}{\mathbf{Z_{EQ}}}\right)\mathbf{Z_X}$$

where the equivalent (total) impedance of the circuit is given by:

$$\mathbf{Z_{EQ}} = \mathbf{Z_1} + \mathbf{Z_2} + \cdots + \mathbf{Z_N}$$

All of the values shown in **bold** are phasors.

2. **Kirchhoff's Current Law and the Current Division Principle:**

Kirchhoff's Current Law (KCL) states that the algebraic sum of the currents at any node in an electric circuit is zero.

In Figure 2 using the convention that the current entering a node is negative and that leaving the node is positive, the KCL equation is given below.

$$-\mathbf{I_S} + \mathbf{I_1} + \mathbf{I_2} = 0$$

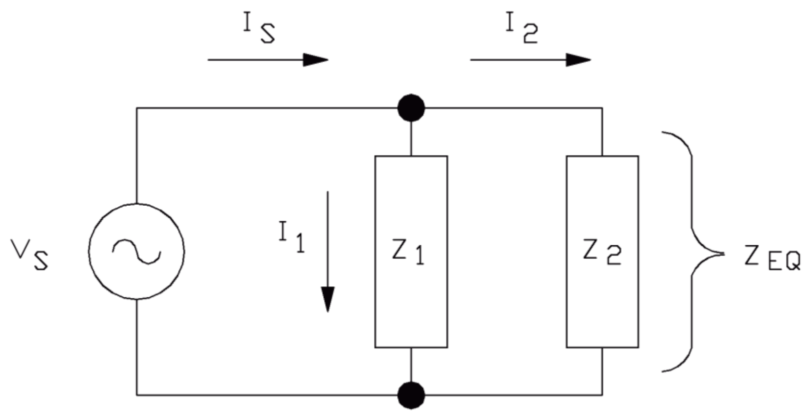


Figure 2
Currents at a node of an AC circuit
(all values are phasors)

According to the current division principle, the current through an impedance in an AC parallel circuit is inversely proportional to its impedance value. The current $\mathbf{I_X}$ impedance $\mathbf{Z_X}$ can be shown to be:

$$\mathbf{I_X} = \mathbf{I_S}\left(\frac{\mathbf{Z_{EQ}}}{\mathbf{Z_X}}\right)$$

where $\mathbf{Z_{EQ}}$ is the equivalent impedance of the parallel impedances $\mathbf{Z_1}$ and $\mathbf{Z_2}$. The general case equation is:

$$\frac{1}{\mathbf{Z_{EQ}}} = \frac{1}{\mathbf{Z_1}} + \frac{1}{\mathbf{Z_2}} + \frac{1}{\mathbf{Z_3}} + \cdots + \frac{1}{\mathbf{Z_N}}$$

In this case there are only two impedances in parallel so

$$\frac{1}{\mathbf{Z_{EQ}}} = \frac{1}{\mathbf{Z_1}} + \frac{1}{\mathbf{Z_2}}$$

which can be simplified to

$$\mathbf{Z_{EQ}} = \frac{\mathbf{Z_1 Z_2}}{\mathbf{Z_1} + \mathbf{Z_2}}$$

3. **Node Analysis:**
In a circuit with N nodes, one node is chosen to be the reference node (“ground”) and an arbitrary voltage (usually zero) is assigned to it. If the voltages at the remaining (N-1) nodes are unknown then KCL equations can be written at these (N-1) nodes and the resulting (N-1) nodes simultaneous equations can be solved for the (N-1) unknown node voltages. Any branch current can then be calculated using **V** = **ZI**. In Figure 3 there are three nodes (S, 1 and 2) other than ground. However in this case the $\mathbf{V_S}$ node voltage is known so we get N=2.

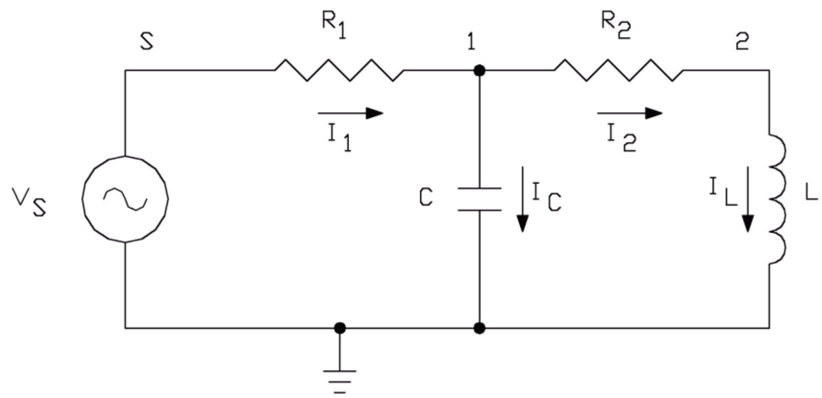


Figure 3
Circuit to explain AC node analysis

At node 1 KCL says that

$$\mathbf{I_1} = \mathbf{I_C} + \mathbf{I_2}$$

therefore

$$\frac{\mathbf{V_S} - \mathbf{V_1}}{R_1} = \frac{\mathbf{V_1}}{\mathbf{Z_C}} + \frac{\mathbf{V_1} - \mathbf{V_2}}{R_2} \quad \text{EQ 1}$$

At node 2 KCL says that

$$\mathbf{I_L} = \mathbf{I_2}$$

therefore

$$\frac{\mathbf{V_2}}{\mathbf{Z_L}} = \frac{\mathbf{V_1} - \mathbf{V_2}}{R_2} \quad \text{EQ 2}$$

solving equation 2 for $\mathbf{V_1}$ gives

$$\mathbf{V_1} = \left(1 + \frac{R_2}{\mathbf{Z_L}}\right)\mathbf{V_2} \quad \text{EQ 3}$$

In Figure 3 let: $\mathbf{V_S}$ = 10 V $\underline{/0^0}$, f = 1Khz, R_1 = 100 Ohms, R_2 = 200 Ohms, L=20 mH, and C = 0.1 µF.

$$\mathbf{Z_L} = j\omega L = j2\pi fL = j2\pi(1000)(0.02) = j125.66$$

therefore EQ 3 becomes

$$\mathbf{V_1} = \left(1 + \frac{200}{j125.66}\right)\mathbf{V_2} = (1 - j1.5916)\mathbf{V_2} \quad \text{EQ 4}$$

$$Z_C = \frac{1}{j\omega C} = \frac{-j}{2\pi fC} = \frac{-j}{2\pi(1000)(1x10^{-7})} = -j1591.55$$

substituting EQ 4 into EQ1 gives

$$\frac{10-(1-j1.5916)\mathbf{V_2}}{100}=\frac{(1-j1.5916)\mathbf{V_2}}{-j1591.55}+\frac{(1-j1.5916)\mathbf{V_2}-\mathbf{V_2}}{200}$$

solving for $\mathbf{V_2}$ gives

$$\mathbf{V_2}=\frac{20}{\left(2+\frac{j200}{1591.55}\right)(1-j1.5916)-j1.5916}=3.889\ \mathrm{V}\underline{/64.68^0}\quad \text{EQ 5}$$

Equation 5 has complex numbers which must be evaluated. Excel can be used to do math with complex numbers. Figure 4 shows an example of the Excel equations used to solve equation 5. In column A are the Excel equations which produce the results shown in column B. Note that in working with complex numbers in Excel the mathematical operators +, -, * and / are not used. You can use the Excel help page to see the correct format for working with the functions COMPLEX, IMSUM, IMPRODUCT, IMDIV and IMABS.

	A	B	
1	-1.5916	-1.5916	
2	=+COMPLEX(0,A1)	-1.5916i	
3	=+IMSUM(1,A2)	1-1.5916i	
4	=COMPLEX(0,200/1591.55)	0.125663661210769i	
5	=+IMSUM(2,A4)	2+0.125663661210769i	
6	=+IMPRODUCT(A3,A5)	2.20000628318306-3.05753633878923i	
7	=+IMSUM(A6,A2)	2.20000628318306-4.64913633878923i	
8	=+IMDIV(20,A7)	1.66323807846035+3.51481750290282i	V2
9	=+IMABS(A8)	3.88848337843336	V2 mag
10	=+DEGREES(IMARGUMENT(A8))	64.6761352594477	degreees
11	=+IMPRODUCT(A3,A8)	7.25742161608048+0.867607777225327i	V1
12	=+IMABS(A11)	7.30909787652718	V1 mag
13	=+DEGREES(IMARGUMENT(A11))	6.81722282554287	degreees

Figure 4
Excel equations used to solve equation 5

The circuit in Figure 3 can also be analyzed using MATLAB to solve the two simultaneous equations (equations 1 and 2) in matrix form. An example of this is shown in the Appendix.

Equipment Required:

- Tektronix TBS 1064 Oscilloscope
- Tektronix AFG 1022 Function Generator
- resistors, inductor and capacitor

Procedure:

1. Construct the circuit shown in Figure 5 on the breadboard. Use the AFG 1022 Function Generator to provide the voltage source, VS. Set the magnitude of this voltage source to 10 V_{PEAK} and set the frequency, f, to 1000 Hz.

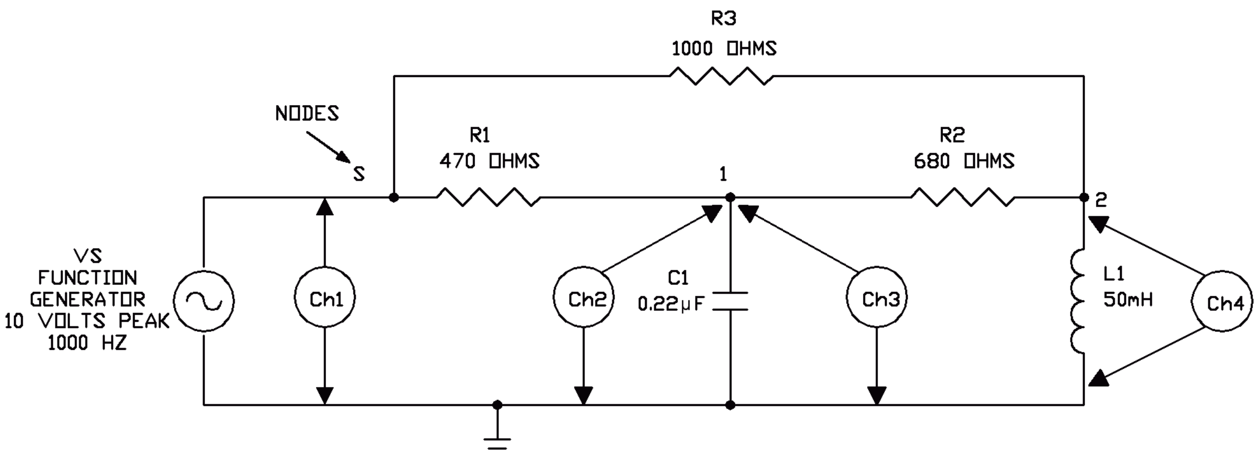


Figure 5
AC circuit to be analyzed

2. Connect Ch 1 of the TBS 1064 Oscilloscope to measure the voltage at node S (the output of the signal generator) with respect to ground. This is the reference voltage which all other voltages will have their phase compared to.
3. Connect Ch 2 and Ch3 of the TBS 1064 Oscilloscope together using short BNC to BNC cables connected to a T BNC adapter. Connect the third connection on the T BNC adapter to node 1. Ch 2 and Ch3 of the TBS 1064 Oscilloscope now both measure the voltage at node 1.
4. Connect Ch 4 of the TBS 1064 Oscilloscope to measure the voltage at node 2.
5. With Ch 1 and Ch 4 being displayed use the voltage cursors to measure the peak to peak waveform voltage of nodes S and 2. Using the time cursors measure the time difference between the rising edges (where the waveforms cross zero volts) of the waveforms at nodes S and 2. Calculate the phase shift using Figure 6. Record these reading here and in your report. Save the display for inclusion in your report.

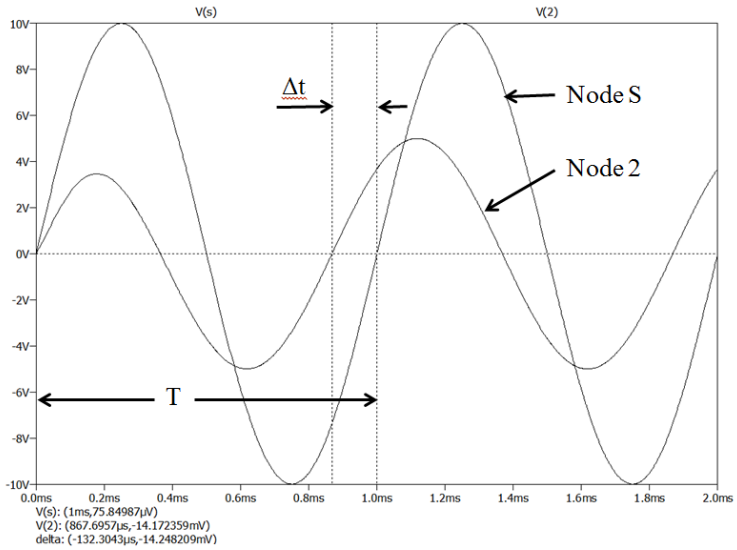
$V_{S\ P\text{-}P}$=_________ $V_{2\ P\text{-}P}$=_________

$\Delta t_{S\text{-}2}$=_________ $\varnothing_{S\text{-}2}$=__________

The time difference, Δt, between Ch 1 (node S) and Ch 4 (node 2), is shown in Figure 6. The phase shift between the measured voltage and the signal generator can be calculated from the equation

$$\varnothing = \text{Phase Shift} = \left(\frac{\Delta t}{T}\right)(360^{o})$$

Where $T = \frac{1}{f} = \frac{1}{1000} = 0.001$ seconds



Note: If the node waveform appears before the node S waveform Δt and Ø are positive.

Figure 6

Measuring Δt

6. With Ch 1 and Ch 2 being displayed use the voltage cursors to measure the peak to peak waveform voltage of node 1. Using the time cursors measure the time difference between the rising edges (where the waveforms cross zero volts) of the waveforms at nodes S and 1. Calculate the phase shift using Figure 6. Record these reading here and in your report. Save the display for inclusion in your report.

$V_{1\ P\text{-}P}$=_________

$\Delta t_{S\text{-}1}$=_________ $Ø_{S\text{-}1}$=__________

7. With Ch 1 and Ch 2 being displayed push the Math button and select CH1-CH2. Turn off Ch2 by pushing the 2 (blue) button. Use the voltage cursors to measure the peak to peak waveform voltage of the Math waveform. Using the time cursors measure the time difference between the rising edges (where the waveforms cross zero volts) of the waveforms at nodes S and the Math function. Calculate the phase shift using Figure 6. Record these reading here and in your report. Save the display for inclusion in your report.

$V_{R1\ P\text{-}P}$=_________

$\Delta t_{S\text{-}R1}$=_________ $Ø_{S\text{-}R1}$=__________

8. With Ch 1, Ch 3 and Ch 4 being displayed push the Math button and select CH3-CH4. Turn off Ch3 and Ch4 by pushing the 3 (purple) and 4(green)buttons. Use the voltage cursors to measure the peak to peak waveform voltage of the Math waveform. Using the time cursors measure the time difference between the rising edges (where the waveforms cross zero volts) of the waveforms at nodes S and the Math function. Calculate the phase shift using Figure 6. Record these reading here and in your report. Save the display for inclusion in your report.

$V_{R2\ P\text{-}P}$=_________

$\Delta t_{S\text{-}R2}$=_________ $\varnothing_{S\text{-}R2}$=__________

9. Disconnect Ch 3 and Ch4 from the circuit and move Ch 2 from node 1 to node 2 as shown in Figue 7.

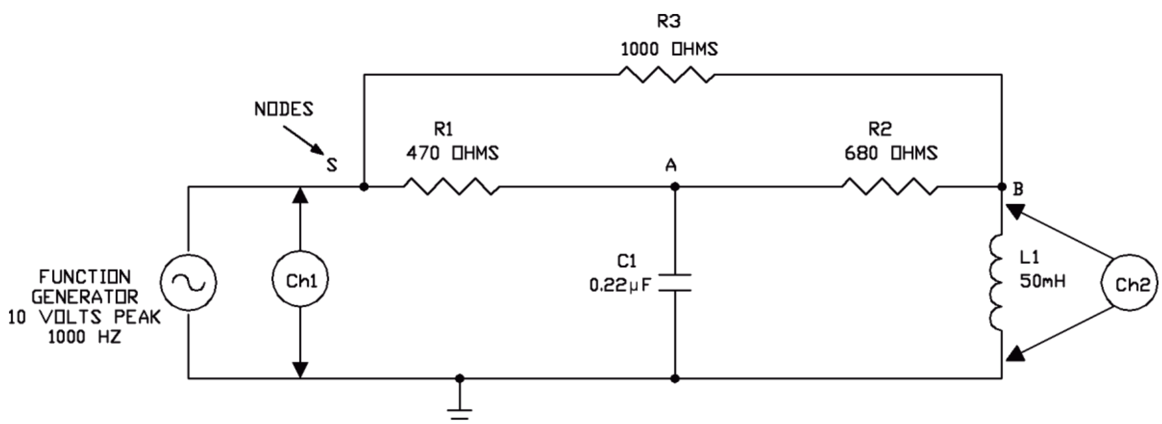


Figure 7
Configuration used to measure the voltage across R3

10. With Ch 1 and Ch 2 being displayed push the Math button and select CH1-CH2. Turn off Ch2 by pushing the 2 (blue) button. Use the voltage cursors to measure the peak to peak waveform voltage of the Math waveform. Using the time cursors measure the time difference between the rising edges (where the waveforms cross zero volts) of the waveforms at nodes S and the Math function. Calculate the phase shift using Figure 6. Record these reading here and in your report. Save the display for inclusion in your report.

$V_{R3\ P\text{-}P}$=_________

$\Delta t_{S\text{-}R3}$=_________ $\varnothing_{S\text{-}R3}$=__________

Report:

The report will be a formal report and must include the following:

1. Calculate the node voltage phasors based on a node analysis of the circuit shown in Figure 5. Resistor, R3, adds an extra term to the node currents at node 2 (see equation 2) and the value of the components are not the same in Figures 3 and 5. The use of Excel or MATLAB is strongly encouraged. Calculate the component currents. Verify that KCL is verified at node 1 and node 2. Show all of your work including the equations which show how you did the node analysis and Excel equations or the MATLAB program file in the report.
2. Based on the measured voltages and phase shifts calculate the current phasors in each component. Use the nominal values for all components. Verify that KCL is verified at node 1 and node 2.
3. Use LTspice or Pspice to simulate the circuit in Figure 5. Just add a "**.**AC list 1000" spice directive to Figure 5 to get an AC solution at 1000 Hz.
4. Include in the report a table comparing the calculated, measured and simulated voltages and currents. These values are phasors so the value includes both the magnitude and phase of each value.
5. A conclusion paragraph to comment on how well the observations met expectations. Point out any significant discrepancies. Give the reason for the discrepancies. Use analytic techniques to do so to the extent that you can. Don't just guess.

Appendix: MATLAB AC Circuit Node Analysis

```
% EE283 Assignment 6 AC Node Analysis
% JB Gilmer   8/15/17  script file EE283L6.m
% Inputs: Parameters of matrices A and b for node analysis (in file)
% Purpose: Find the Node Voltages and branch currents of circuit
% Outputs: Node Voltages V1 and V2, Currents I1, I2, IC, IL
format short g
format compact
% Definition of circuit parameters
f=1000; %Hz
R1=100; %Ohms
R2=200; %Ohms
ZC=1/(j*2*pi*f*.1e-6); %Ohms  C=0.1uF
ZL=j*2*pi*f*20e-3;  % Ohms    L=20mH
VS=10.0; %Volts (at 0 degrees)
%Circuit
%        |---\/\/\/---V1---\/\/\/---V2
%        |    R1       |      R2     3
%       VS             = C           3 L
%        |             |             |
%       Gnd           Gnd           Gnd
%Equations
% EQ1  -(VS-V1)/R1 + V1/ZC + (V1-V2)/R2 = 0
% EQ2  -(V1-V2)/R2 + V2/ZL = 0
% A and b matrix parameters for Ax=b  x=[V1 V2]'
a11= 1/R1+1/ZC+1/R2;
a12= -1/R2;
a21= -1/R2;
a22= 1/R2+1/ZL;
A=[a11 a12;a21 a22];
b=[10/R1 0]';
%Solve for V1 and V2
x=inv(A)*b;
V1 = x(1)
V2 = x(2)
%Find branch currents
I1 = (10-V1)/R1
I2 = (V1-V2)/R2
IC = V1/ZC
IL = V2/ZL
%Find node net currents
IN1= -I1+I2+IC
IN2= -I2+IL
```

Execution in MATLAB session:

```
>> EE283L6
V1         7.2575 +     0.86763i
V2 =        1.6633 +      3.5149i
I1 =      0.027425 -   0.0086763i
I2 =      0.027971 -    0.013236i
IC =  -0.00054515 +     0.00456i
IL =      0.027971 -    0.013236i
IN1 =  1.0083e-017 +8.6736e-019i          (This is close to zero)
IN2 =            0 +1.7347e-018i          (This is close to zero)
```