

EE283: About Lab 2

Lab #2 concerned meter impedance (and how that affects measurements) and maximum power output. In this document, I'd like to focus on the meter impedance problem, because that's where the surprises were. The basic scenario is that a DMM (high impedance) and a VOM (low impedance) are used to measure the same node in a simple circuit, shown in at right.

If a Voltmeter is ideal, the meter resistance R_M is infinity. So, point V_A is the result of a simple Voltage divider.

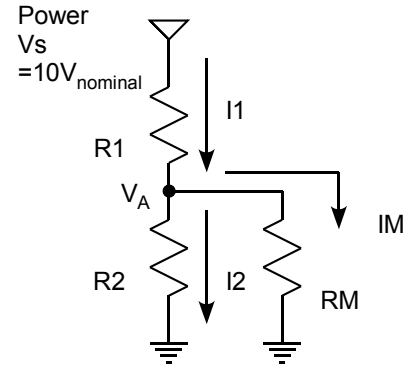
If $R_1 = 1M\Omega$, and $R_2 = 300K\Omega$, then V_A must be 10V ($R_2/(R_1+R_2) = 2.31V$). (I give this Voltage to only 3 digits because the R values and V_S are only good to three digits.) The current through each resistor must be equal, and that comes out to $I_1=7.7\mu A$ and $I_2=7.7\mu A$ (when done to two significant figures). All looks nice and happy, right?

There are two different problems we try to address with this circuit. The first is, what is the effect of having non-ideal meter impedance R_M ? This isn't hard to understand. If R_M is actually $10M\Omega$ (DMM) then that $10M\Omega$ resistor in parallel with R_2 means that the Voltage divider needs to be recalculated with $R_2 \parallel R_M = 291K\Omega$ substituted for the value of R_2 . That gives us $V_A = 10V (291K\Omega/(1MW+291K\Omega)=2.26V$. There is a measurement error of $-0.05V$. Not a whole lot, but perhaps significant in some cases. If instead we use a VOM, R_M is smaller. Let's use $20K\Omega/Volt$ (I think that's the spec for the orange VOM's in the lab). So, if we are using the 10 Volts DC scale, V_M should be $200K\Omega$. That in parallel with R_2 gives us $120K\Omega$. Quite different from the ideal of $300K\Omega$. So, when we measure V_A , we see $V_A = 10V (120K\Omega/120K\Omega+1M\Omega)=1.07V$. That's a huge error!

Now, if we were to do the same thing with the DMM and $1K\Omega$ and 300Ω resistors (instead of $1M$ and $300K$), we'd find almost now error. Error would be pushed out three orders of magnitude; we wouldn't see it. ($10M\Omega$ in parallel with 300Ω is still 300Ω .) Even if we use the VOM, our measurement is not bad: V_B (I'll call it) = $10V (.299K\Omega/(1.0K\Omega+.299\Omega)= 2.30V$ We have an error of just $.01V$, which is below the visual resolution of the meter (probably only good to about $.03V$ in this part of the scale).

So, the point is, the high impedance circuit (with $R_1=1.0M\Omega$ and $R_2 = 300K\Omega$) is hard to measure accurately, and even the DMM will have a bit of error, while the VOM is hopeless. Whereas, with low impedances around $1K\Omega$ or less, both instruments do fine. (The DMM can read more digits, but if there is any AC riding on the DC it may be fooled or give odd readings, while the DMM will likely give better results. There are some times and places where an analog meter is better. And, it is possible, with electronics, to build high impedance analog meters too. They just won't be cheap.)

The second problem is more difficult. We want to use the same circuit to measure the meter impedance. The very fact that a high impedance meter doesn't affect the circuit much makes it hard to accurately find its impedance! After all, the fact that it is there at all is very hard to discern if you are looking for a Voltage change. (Of course, you can't see the actual Voltage without it! So the ideal has to be calculated.)



If you look at the R_M equation, it's not easy to see what is going on:

$$R_M = (R_1 R_2) / (R_2 (V_S/V_A - 1) - R_1)$$

Look at the denominator. When you plug in numbers, especially with the VOM or the 1K Ω /300 Ω resistor pair, that final subtraction of R_1 gives a number that is the difference between two very close large numbers, and the difference is very small. It may even result in a negative denominator! Very small changes in V_S or V_A can make a big difference in the final R_M value. What does it mean? Well, it means that it's hard to get a good R_M measurement.

Here's another way to look at it. Let's suppose we know the resistor values and Voltages. I'm going to use a set of data from one student on lab 2:

$$R_3 = .998524\text{K}\Omega, R_4 = .29707\text{K}\Omega, V_S = 9.95\text{V}, V_{B(\text{DMM})} = 2.29\text{V}$$

Calculating the current through the resistors, we get:

- The (nominally) 1.0K Ω resistor: $I_3 = (9.95\text{V} - 2.29\text{V}) / .998524\text{K}\Omega = 7.671323\text{mA}$
 - The (nominally) 300 Ω resistor: $I_4 = (2.29\text{V}) / .29707\text{K}\Omega = 7.708621\text{mA}$
- (Notice that I'm carrying extra digits in the middle of my calculations to avoid round-off error. I can, and should, drop them later when I present the results in a table.)

Here's where things get interesting. To two decimals, the currents match. To three decimals (or more) they don't. The difference has to be the current that flows through R_M . What is the current? We quickly do a node equation for node V_B and get $I_3 = I_4 + I_M$. Right? We can solve that for I_M . $I_M = 7.671323\text{mA} - 7.708621\text{mA} = -.03730\text{mA}$. Minus!?!?

A negative value for current means that the I_M is pumping current into node V_B instead of letting it leak out to ground. That means it is acting as a power SOURCE! The Power (V times I) is negative. That means instead of dissipating energy, it is injecting energy. If we divide the Voltage by the current, we get a negative R . That means the higher the Voltage the meter sees, the more current it pumps in. That is NOT normal meter behavior. But that's what a negative R means. So that leaves us with a couple of possible hypotheses concerning this circuit:

Hypothesis 1) Yeah, the meter really is pumping current into the node V_B . We can test this. Check with a different DMM. For the VOM, they actually do include a battery to be used for measuring resistance. It provides a Voltage source and the meter measures current through the component and displays resistance on a scale that shows how much resistance would give that current. Could that battery be leaking juice into the Voltage measurement? Not hard to check – just remove the battery. With the DMM that's harder (Impractical) to do. If we were to do this, we would deny this hypothesis – our readings would likely be the same. Let's set this possibility aside and consider:

Hypothesis 2) There is something wrong with a measurement. We can check this too. Which reading is likely to be in error? We have four different ones to choose from.

Let's start with the V_B measurement itself. Suppose V_B measured just .01 Volt lower. Calculating the current through the resistors again, we get:

- The (nominally) 1.0K Ω resistor: $I_3 = (9.95\text{V} - 2.28\text{V}) / .998524\text{K}\Omega = 7.681338\text{mA}$
- The (nominally) 300 Ω resistor: $I_4 = (2.28\text{V}) / .29707\text{K}\Omega = 7.674959\text{mA}$

And $I_M = 7.681338\text{mA} - 7.674959\text{mA} = .006379\text{mA}$. Solving for $R_M = 357.4\text{K}\Omega$.

Folks, that is plausible. If the meter might be off as much as .01 Volt. That's the difference between a negative R_M and one that is way lower than the actual meter spec R_M of $10M\Omega$. The point is, a very, very small change in this one particular variable can change the outcome of our meter impedance calculation enormously! Even for the VOM, that's true. We should read as many digits of accuracy as we can. Maybe if the Voltage measurements were made to another digit or even two of accuracy we'd have not seen a negative resistance.

This Voltage is not the only variable that can change. Where did you measure the power supply Voltage? Those actual wires are not completely resistanceless. Between the power supply and the resistor R1 or R3 there can be an Ohm or more. Enough to matter. I looked up the spec on these common resistors, and for smaller ones below $150K\Omega$, the spec (if I read it right) says that a $1K\Omega$ resistor might change + or - up to 3.5 Ohms from a $10^\circ C$ temperature change. With a $1K$ Ohm resistor with $7.7mA$ through it, that's $59mW$, or 24% of the component's rated dissipation of $1/4W$. So, it could be getting hotter than just up $10^\circ C$. That could matter! It's still easily within its 5% tolerance, but your measurement of R_M goes whacko.

So, if you are trying to measure the meter resistance, you want a circuit where the presence or absence of the meter makes a big difference, not a small difference. In this lab exercise, that's the V_A circuit. Better yet, just put the meter in series with a $1.0M\Omega$ resistor, and see what Voltage it says it sees from a 10 Volt supply. The DMM will say pretty close to 9.09Volts, since it is a $10M\Omega$ resistor in series with a $1M\Omega$ resistor. (Essentially, we make R2 infinity.)

Now, concerning the lab. If you got a surprising resistance, especially a negative one, for R_M , what do you do? I was surprised to see students not even mention it. Or, cover it up by changing the sign to positive, perhaps hoping it would not be noticed. No! It's interesting. It's a surprise! And, it's a learning opportunity. When something does not go as expected, the worst thing you can do is hide it. You are practicing to go out and be engineering professionals. If you cover up problems, things will be much worse when they eventually blow up. Then they will start looking for who to blame. Better to call someone's attention to a problem early when something may be done to fix it, or at least mitigate the damage. Or, you may find you have discovered something useful, even productive. (Like a perpetual motion machine that creates energy from nothing, like a negative resistor? Ahem. There are laws against that.)

We do things a bit different in engineering than what you may be used to. In science labs, the last thing they want is for you to have something unexpected happen. The theory is covered first in detail, then the experiments are constructed with fine (often expensive) components that won't fail to give you the right results if you just follow instructions. If there's something unexpected, you assume it's your fault. And, you see your job as getting a good grade, so you cover it up if you can. That's not how we do things. We want you to see the difference between the theory and models and the real world, which is often messy, imprecise, and, yes, sometimes troublesome. We want you to get used to seeing surprises. When that happens, get curious. Try to figure it out. It's fun and interesting. If you are going to build things, especially things that nobody has built before, there are going to be surprises. Sometimes things won't work. But that's how you will learn, and occasionally make discoveries or gain important insights. This lab exercise was a little step along that path.