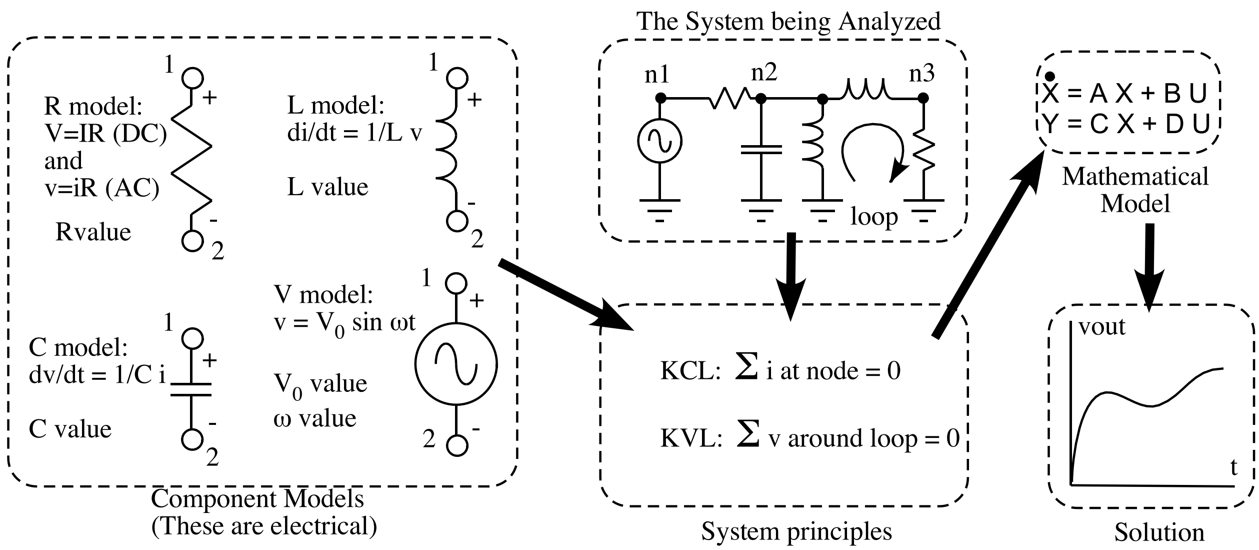
EGR222 Mechatronics: Notes Concerning System Modeling

These notes are intended for supplemental use for EGR222, as the focus of the class moves from individual components of mechatronic systems (sensors, actuators, motors, etc.) to the aggregate behavior of systems made up of those components. These revised / annotated notes will begin with an overview of what we want to accomplish. For the sake of simplicity, let's consider an electric – only system, as shown in the figure below



What we typically want to do, and we have done this for electrical systems previously (EE211 for example), is to analyze the circuit by building a mathematical model of it so that we can predict its behavior. We start with a circuit. That defines various components, and also prescribes how those components are connected, the "topology" of the circuit. For each component, we have a "component model" which describes how it behaves. In this simple example, all of the components are "One Port" components – they have two terminals, one "port", with which they interact with the rest of the circuit. Each component equation defines the relationship between the circuit quantities between the two terminals of the component, which we usually label as #1 and #2 or positive and negative for purposes of tracking sign. (Notice that for the Voltage source, Voltage is defined and current is left undefined; it will be a consequence of the rest of the circuit.) We can also call upon other "circuit" principles – Kirchoff's Current and Voltage laws, that define the behavior of circuits. Using those equations, and substituting into them the component equations in a systematic manner, we can eventually get simultaneous differential equations that can be solved together in matrix form.

The exact same principles hold true for mechanical systems as well! The components and the component equations are different. And yet, they are the same! The "circuit principles are different. In the mechanical world, you do a Force balance at a point. But, that's really no different than Kirchoff's Voltage law! It's just expressed differently, uses different units, and looks different. So, when a mechanical system is solved, the process is very much like what is outlined above for an electrical system, whether it is mechanical translation, mechanical rotation, hydraulic, pneumatic, or even heat. (Well, heat has nothing that corresponds to inductance.)

Even more important, we can build a combined model for a combined system. That's where we are headed.

So, now let us examine electrical, mechanical translational, and mechanical rotational systems and components. (This is in the book as a table, and in my raw notes, but it needs elaboration.) I'm going to break up the table and consider it a piece at a time:

Fundamental Position Concepts

	Electrical	Mechanical	
		Translational	Rotational
Position	q (charge, Coulombs)	x (position, in cm / m/ in...)	θ (rotary position, ° or rad)
Velocity	i = dq/dt (Amperes)	v = dx/dt (cm/sec, mph, etc.)	ω = dθ/dt (rad/sec, RPM)
Acceleration	di/dt	a = dv/dt (cm/sec^2, etc.)	dω/dt

So, in all these different domains (electrical, or mechanical translational or rotational) we have some notion of "position". In a mechanical system, this is perhaps the most easily observed attribute of the system; one can literally see it. A mass, or the terminal of a spring, or something else has some observable position. One usually marks a reference "zero", often the position of whatever it is when the system is "at rest" (or storing no energy). For an electrical system, the concept of position is a bit more abstract. One can imagine that every point in the circuit has some stray capacitance on which "charge" is stored. Charge is best thought of as excess or a deficit of electrons, aggregated together to give some number in units of "Coulombs". It's difficult to directly observe charge, which is one reason electrical behavior is more difficult to understand; one can't see the stuff that moves.

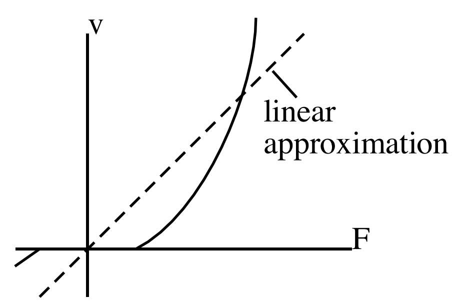
It is when things move that systems get interesting. In a mechanical system, a rod might move from left to right, "x" in some frame of reference. Or a shaft may rotate. In an electrical system, the electrons move. Or, synonymously, the "charge" moves, called "current". That is observable with instruments called Ammeters, that use the magnetic field exerted by moving charge as a means of detection. In the mechanical domains there is no special unit for movement. It's just the derivative of position with respect to time. But in the electrical domain, "charge" is only rarely used as a unit; Amperes is much more commonly used as the attribute of flow, of stuff moving, as if it was the fundamental measurement of electrical position. So, here we see that even though the basic concepts are fundamentally the same between the electrical and mechanical domains, there are practical differences in the way systems are characterized and measured.

All three of these domains have the equivalent of not only "position", but also the notion of "Force." In the mechanical world, that is what causes stuff to move. The same is true in the electrical domain. We call it an "Electromotive Force" (or e.m.f.) or Voltage. Together with position, this gives us the tools to consider "steady state" systems. By "steady state" we don't mean that nothing is moving, it means that velocities are unchanging. (A "static" system, like a bridge, is one in which there is no movement at all.) In a steady state system, forces are constant, as are velocities. A motor driving a fan, or a DC circuit, are examples.

Fundamental Force Concepts

	Electrical	Mechanical: Translational	Mechanical: Rotational
Force	V (Voltage, in Volts)	F (Force, N, $grams_f$, dynes..)	T (Torque, N-m, g-c, d-c,..)
Friction(R)	R (Resistance, Ohms=V/A)	F/v (Force per unit of velocity, e.g. g_f/cm/sec)	T/ω (Torque per RPM, etc.)
Work Done	V•I•Δt (Watt-sec, Joules)	F•Δx (Work, N-m, J. etc.	T•ω (g-cm-sec, etc.) (watch out! units are tricky!)
Dissipated Energy	P = V•I (Power, Watts, J/sec)	P = F•v (power, J/sec or N-m/sec	P=T•ω (g-cm-sec/RPM, etc.)

This brings us to the most fundamental component concept: a "Resistor". A resistor limits the velocity at which stuff moves under the influence of a force. For electrical resistors, we write "V=IR", Ohm's Law, which is a very good approximation to what actual physical resistors do. If one varies the Voltage, the force causing the electrons to move, the flow rate of those electrons, in Amperes, stays pretty much proportional to the Voltage. That ratio, Resistance, is measured in Ohms. In a DC circuit, the only components we have are resistors and Voltage sources (or, perhaps on rare occasions, current sources). Using Ohm's Law for components and either KCL or KVL circuit equations, we can solve DC circuits.



Mechanical friction is not so well behaved. It is often quite nonlinear. Indeed, there is usually some amount of force, a threshold force, that is necessary to get something moving at all. This presents a problem to modeling. If the motion is steady state, one could find the ratio between velocity (translational or rotational) and Force or Torque, and use that as mechanical resistance. The problem is that the characterization won't be quite right under other conditions. At an example of the variation seen, the motors used for EGR222 were found to have a frictional resistance of about .02 g-c/RPM at 12 Volts, but about .05 g-c/RPM at 5 Volts and .07 g-c/RPM at 3 Volts! (These data were seen in student lab reports for Lab 6.) If we want a model that will be useful over a wide range of conditions, especially if the system might operate in either direction, the simple linear friction model makes sense. But, it needs to be used with the understanding that the model is of limited accuracy.

Motion under Force does "Work". That's Energy. Voltage times Current is Force times rate of movement, Power. To get the energy dissipated, we must specify a time interval. In the mechanical world, we often think of energy first, then find the work per unit time to get power. In all three domains, it's the same: Force times flow. But doing this in the rotational domain is very tricky. Units must come out right. Conversion of RPM to radians is usually needed.

Fundamental Energy Storage Concepts

	Electrical	Mechanical: Translational	Mechanical: Rotational
Mass	Inductance L (Henries L di/dt = V)	Mass: F = ma (grams, kg)	J (or I) (Rotational Inertia) T = J dw/dt (gm-cm/RPM/sec)
Kinetic energy	$E = 1/2\ L\ I^2$ (Joules)	$E = 1/2\ M\ v^2$ (Joules)	$E = 1/2\ J\ \omega^2$ (ω rad/sec)
Spring effect	Capacitance C (Farads; Q = CV)	Spring: F = -kx (grams$_f$/cm)	Torsion spring: T = -kθ (g-c/degree)
Potential Energy	$E = 1/2\ C\ V^2$ (Joules)	$E = 1/2\ kx^2$ (Joules, with unit conv.)	$E= 1/2\ k\ \theta^2$ (Joules, with unit conv.)

AC circuits which have only resistive components, or other components that do not store energy, behave the same way as DC circuits. The same is true for corresponding mechanical circuits, but for the difficulty of building systems with no mass. That's like building electrical circuits with no capacitance or inductance. It can't be done, but there are practical systems for which mass, inductance etc. are of negligible concern.

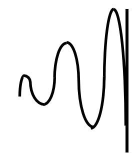
But, for systems with energy storing components, interesting things happen. At some times during operation, a component may absorb energy from the system, and at other times may release that energy. This is perhaps best illustrated by the eccentric orbit of a satellite. When at maximum distance from the Earth, the satellite has maximum potential energy, and moves relatively slowly. As it swings closer to perihelion, it surrenders that potential energy for kinetic energy, which is highest as it swings through its lowest altitude, only to decrease as that energy is converted back to potential energy. In an orbital system, all of that energy is a property of the satellite. But in other electric, electro-mechanical, or mechanical systems energy can be exchanged between different components. For example in a resonant circuit, energy is continuously exchanged between the inductor and capacitor in the circuit.

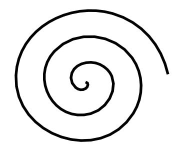
Mass, or its rotational equivalent, rotational inertia, gives motion the property of "storing" kinetic energy. The larger the mass, the greater the energy. Mass is just the resistance of movement to Force, expressed famously by F=ma. In a rotational system, the movement is cyclical, allowing the system to store kinetic energy without displacement, which is in practice quite useful. A "flywheel" is the prototypical rotational mass, giving inertia to discrete cycle engines. The electrical equivalent of mass, inductance, can be thought of as giving the property of momentum to the current passing through it. Just as it takes a good bit of force to slow and stop a heavy moving mass, it takes considerable Voltage to reduce and stop a high current in an inductor. To stop that current instantaneously would require an infinite Voltage, just as stopping a mass instantaneously requires infinite force.

A spring stores energy statically, without requiring motion. The more the spring is compressed (translationally or rotationally), the harder it pushes back. The effect can be reversed, with tension. The same is true for electrical capacitors. The accumulation of current, charge, causes a force, Voltage, that pushes back against further incoming current.

There is an interesting difference between the electrical spring constant, "Capacitance", and the mechanical spring constants. The mechanical spring constant k is given in force per length (or angle). This makes sense. One can easily observe position, and from that deduce the force, which is not so visible, and the corresponding energy stored. In contrast, "Capacitance" is a measure of the amount of medium (charge, or electrons) stored per unit of force, Voltage. This makes sense too. Voltage is easily measured, charge much less so. But it is a point of inconsistency between the mechanical and electrical domains. Yes, Capacitance is like a mechanical spring constant, but it's more precisely like the reciprocal of a spring constant.

It should also be mentioned that, in the mechanical domain, it is practical to craft springs with nonlinear force versus distance characteristics. A spring with the conical form shown at right would increase in stiffness with compression as the outer part of the spring bottoms out against the mounting plate, resulting in a k value that varies with position (or, an exponent other than one for the distance variable). That does not happen to a significant extent in capacitors, though a battery could be viewed somewhat as a nonlinear capacitor.

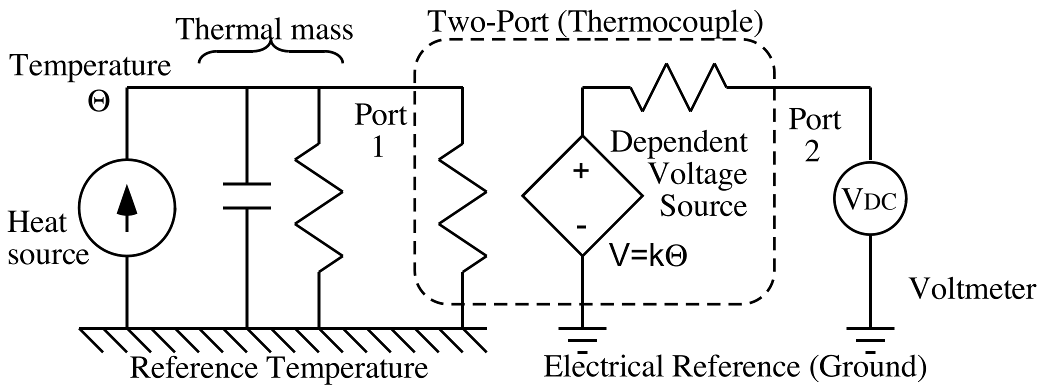




So, the point here is that all of these mechanical and electrical components, representing resistance, a mass effect (kinetic energy storage) or a spring effect (potential energy storage) are mathematically identical, except for some minor units differences and constant placement. We can use the same formulations to solve all three, and in addition systems in other domains such as hydraulics and pneumatics. The basic "circuit" principles apply to all three as well. Kirchoff's Loop equations are the equivalent of force balance. Kirchoff's Current equations are equivalent to conservation of mass. We can use the same symbols for all of these systems. We can use the same modeling tools, such as PSpice, to build mechanical models as well as electrical models. Most importantly, we can use these techniques to build integrated models that represent systems, mechatronic systems, that span multiple domains.

The question remains, "How do we represent a system containing both mechanics and electrical parts as a whole, when each component is distinctly either electrical or mechanical, and the forces and fluids involved are likewise distinct?" The answer is that we have not yet finished filling out our kit of parts. We need a new kind of component called a "Two Port." The components described so far are all "One Ports." They have two "terminals" (in an electrical system), or the equivalent (a position in reference to an origin, for example) in a mechanical system. The state of the component is expressed by the Force / Voltage and Position or Flow (Position or velocity, or Current) for the component. Both the force and flow are in the same domain.

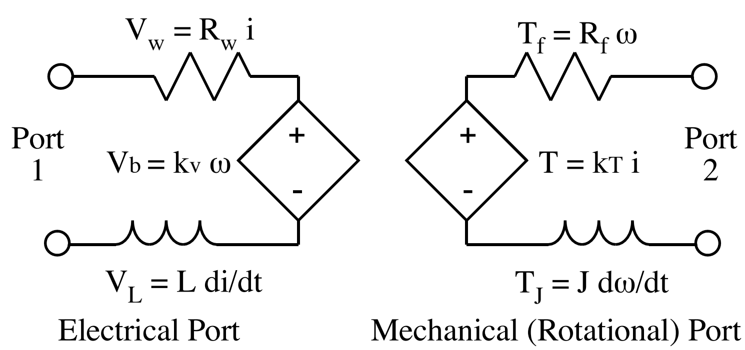
Here's the key reason for a Two-Port: The two ports can be in different domains. A sensor's characteristics at one port can depend on the state of the other port. For example, a thermocouple Voltage depends on the difference in temperature between the "Hot junction" and the "Cold junction" (reference junction). There is a small amount of heat flow along the thermocouple wires. As it appears on the thermal side, it is a heat flow device. The tiny heat flow depends on the temperature difference. That heat flow is converted into a correspondingly tiny electromotive force in the millivolt range. From the electrical port, the device looks like a dependent Voltage source in which the Voltage depends on temperature.



This two port model is part of a relatively simple system. On the thermal side (at left), a heat source pumps heat (Q, thermal energy) into the system, resulting in a rise in temperature (Θ). The thermal mass (Capacitance) allows some heat to leak away (Resistance), so that if the flow of heat ceases, the temperature would go down and eventually follow an exponential decay path to zero (the reference). There is also a resistance path through the thermocouple, but that resistance is "big" and can be neglected for practical purposes when analyzing the thermal part of the system. On the electrical side, the dependent Voltage source supplies current that allows the Voltmeter to read the Voltage. The current drawn by the Voltmeter is small enough that the Thevenin equivalent resistance of the two-port's electrical port, Port 2, has negligible effect.

Another example of a two-port is the motor model we have been using. The figure below shows the model. We see electrical sub-components on the left, and mechanical subcomponents on the right, rendered using electrical symbols. (We could as easily render the right side of the figure with mechanical symbols having the same meanings.) To each port would be connected other components of the appropriate type, such as a Voltage source on the left, and a fan represented as a mechanical resistance on the right, across the respective port terminals of the motor.

PM DC Motor Two-Port Model



The key component of the two-port is the dependent sources. This one happens to be Force sources (Thevenin equivalent) on both sides. But, both sources could be flow sources (Norton equivalent), or there could be one of each. In any case, we can still use our respective system behavior principles, Kirchoff or Force balance, etc. to formulate the part of the system in each domain, and combine these ultimately in a comprehensive system of equations with which the whole system can be solved.